a time which was ripe for such discoveries, because of the movements to make algebra abstract and to algebricize topology.

What started as a problem became a theory and this led to problems again: What are the groups of cohomological dimension one? (By Stallings and Swan, just the free groups). What is the full algebraic interpretation of $H^4(G, A)$ (still a mystery)? Is Whitehead's conjecture true? If $\mathrm{Ext}(G, \mathbf{Z}) = 0$ for given G and the abelian group $\mathbf{Z}$, is G free? (answer, by Shelah, maybe yes or maybe no, depending on your set theory (see Ecklof [1976])). There appears to be a movement in mathematics from problem to ideas to theories to problems to counterexamples—and back again.

Are there breakthroughs of complete novelty? Not quite. As we have argued, there are decisive papers, like the 1942 paper of Hopf which started our whole subject. There were striking new ideas in that paper, but they were not unprecedented; rather, they were rooted, as we have noted, in earlier studies on homotopy and on the homology of Lie groups. Hopf's paper was a new idea, but one built on an older one, hence not a new paradigm. With such a new idea, other developments, here the higher dimensional cohomology, became inevitable—as their multiple discovery shows. In this case, the development came soon; that is not always so, as witness the long wait before the "inevitable" development of the notions of adjoint functors. With the inevitable developments, there are also some which are evitable: They were not needed and they don't seem to matter. It is well known that there are many such papers; just by way of a constructive existence proof, I cite the 1947 paper by Eilenberg and Mac Lane in which the higher cohomology groups $H^n(G, A)$ were interpreted by non-associative multiplications. This result seems to have found no use; no matter, the exploration of the unknown is sure to lead us up some false paths.

Finally, our small piece of history shows that the development of mathematics is by no means single-minded. It involves the interaction between the ideas of many individuals and the interpenetration of different fields. In the present case, the interplay between algebra and topology is prominent, and is typical of the contributions of Beno Eckmann to our science.

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Saunders Mac Lane

Department of Mathematics
University of Chicago
Chicago, Illinois 60637
USA

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