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A COINCIDENCE-FIXED-POINT INDEX [1]

by Albrecht DOLD

B. Eckmann anlässlich seines 60. Geburtstages gewidmet

INTRODUCTION

The fixed point set of a map $\varphi: X \to X$ is, generically, a discrete set; if it is compact its (weighted) cardinality is measured by the Hopf-index $I(\varphi) \in \mathbf{Z}$. The coincidence set K of a pair of maps $(\varphi, p): X \rightrightarrows Y$ is not discrete; its generic dimension is $\dim K = \dim X - \dim Y$. If K is compact it can sometimes (compare 3.8) be measured by a cohomology invariant κ, but even then κ is difficult to deal with. This might explain why most studies on coincidence questions make additional assumptions on (φ, p), or use auxiliary data. For instance, if one of the maps, say p, admits a section of sorts σ then the fixed points of $\sigma\varphi$ are in K so that fixed point methods give coincidence results. Usually σ is not a genuine section; for instance, if p is a Vietoris map then one uses $(p^*)^{-1}$, on the cohomology level (cf. 3.7).

The idea of the present lecture is to let fixed point transfers in the sense of [2] play the role of σ; we have to assume, therefore, that p is ENR_Y which means (roughly speaking; cf. [2]) that p has sufficiently many local sections. Actually, our procedure for counting fixed points of $\sigma\varphi$ (cf. §1) is much more elementary than [2] and doesn't really use transfers. Only when we express the number of fixed points of $\sigma\varphi$ as a Lefschetz trace in theorem 2.1, transfers t become essential. If one imposes further (rather restrictive) assumptions on p then t can be eliminated again (from the theorem; it is still used in the proof), as shown in prop. 3.5. — The last section of the paper discusses applications (3.1-3.6) and problems (3.7, 3.8).

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[1]) Presented at the Colloquium on Topology and Algebra, April 1977, Zurich.