

**Zeitschrift:** L'Enseignement Mathématique  
**Band:** 24 (1978)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** ON A FUNCTIONAL EQUATION RELATING TO THE BRAUER-  
RADEMACHER IDENTITY

### **Bibliographie**

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**DOI:** <https://doi.org/10.5169/seals-49690>

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and

$$R(p^\beta, p^\alpha) = \mu_A(p^{st}) \sum_{d \in A(p^{jt})} f(d) h\left(\frac{n}{d}\right) = 0, \text{ since } s \geq 2.$$

Now, it is easy to see that  $L(p^\beta, p^\alpha) = R(p^\beta, p^\alpha)$  if and only if (16) holds. Thus the Theorem is proved.

**COROLLARY** *If  $f \in \mathcal{C}$  and  $h = \mu_A g$ , where  $g \in \mathcal{M}$ , then the pair  $(f, h)$  satisfies the functional equation (14).*

*Proof.* We have

$$\begin{aligned} F_A(p^\alpha) f(p^{\alpha-t}) &= \left\{ \sum_{d \in A(p^{st})} f(d) \mu_A(p^{st}/d) g(p^{st}/d) \right\} f(p^{(s-1)t}) \\ &= \left\{ \sum_{i=0}^s f(p^{it}) \mu_A(p^{(s-i)t}) g(p^{(s-i)t}) \right\} f(p^{(s-1)t}) \\ &= \{ f(p^{st}) - f(p^{(s-1)t}) g(p^t) \} f(p^{(s-1)t}) \\ (17) \quad &= f(p^{(2s-1)t}) - f(p^{(2s-2)t}) g(p^t), \end{aligned}$$

since  $f \in \mathcal{C}$ . Also,

$$\begin{aligned} f(p^\alpha) F_A(p^{\alpha-t}) &= f(p^{st}) \left\{ \sum_{i=0}^{s-1} f(p^{it}) \mu_A(p^{(s-1-i)t}) g(p^{(s-1-i)t}) \right\} \\ &= f(p^{st}) \{ f(p^{(s-1)t}) - f(p^{(s-2)t}) g(p^t) \} \\ (18) \quad &= f(p^{(2s-1)t}) - f(p^{(2s-2)t}) g(p^t). \end{aligned}$$

Now, from (17) and (18), we see that (16) holds for all primes  $p$  and all integers  $\alpha \geq 2$ , where  $t = \tau_A(p^\alpha)$ . Hence the Corollary follows in virtue of the Theorem.

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(Reçu le 9 juin 1977)

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