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THEOREM 3. (Guillemin [10], Losik [17]). $C_{\Delta}^*(L_M, \Omega_M)$ is a model for a bundle E with fiber F_n , base space M , associated to the tangent bundle of M .

More precisely, a model for $C_{\Delta}^*(L_M, \Omega_M)$ is the DG-algebra $\Omega_M \otimes WU_n$ over Ω_M , where

$$d(1 \otimes c_i) = 0 \quad d(1 \otimes h_i) = 1 \otimes c_i - p_{i/2} \otimes 1$$

where $p_{i/2}$ is zero if i is odd and is a form representing the Pontrjagin class of M of degree $2i$ if i is even.

Note that if a foliation F on $X \times M$ transverse to the fibers $\{x\} \times M$ is given, one has a characteristic homomorphism

$$C^*(L_M, \Omega_M) \rightarrow \Omega_{X \times M}$$

One has also a morphism

$$WO_n \rightarrow C_{\Delta}^*(L_M, \Omega_M)$$

(or $WU_n \rightarrow C^*(L_M, \Omega_M)$ in case M has trivial Pontrjagin classes) whose composition with the previous one is the usual characteristic homomorphism for the foliation F (cf. [3], [12]).

4. MAIN THEOREM

THEOREM 1. $C^*(L_M)$ is a model for the space Γ of continuous sections of the bundle E described in the theorem above.

This result, first conjectured by Bott (and also Fuks), has been proved by several people (Bott-Segal¹⁾, Fuks-Segal, Haefliger [13], Ph. Trauber, and others).

Suppose that G is a compact connected Lie group acting on M . Then it also acts on the bundle E and on its space of sections. Let us denote by Γ_G the total space of the bundle with fiber Γ associated to the universal G -bundle with base space BG .

THEOREM 1'. $C^*(L_M; G)$ is a model for the space Γ_G .

The way I proved theorem 1 was to construct first a tentative algebraic model A for Γ following ideas of R. Thom [20] and D. Sullivan [18], and

¹⁾ Added on proof: *Topology* 16 (1977), pp. 285-298.

a morphism of A in $C^*(L_M)$. Then one proves directly that it induces an isomorphism in cohomology. The fact that A is also a model for Γ was proved in a similar way (cf. [14]).

When M has a finite dimensional model, one can construct a model for Γ which is finite dimensional in each degree, and with it one can make explicit calculations.

Note that the inclusion $C_{\Delta}^*(L_M, \Omega_M) \rightarrow C^*(L_M, \Omega_M)$ is a model for the evaluation map $\Gamma \times M \rightarrow E$ associating to a section s and a point x of M the element $s(x)$ of E .

For computations along the lines of the spectral sequence of Gelfand-Fuks, see Cohen and Taylor [22].

The proof of theorem 1' is very similar to the proof of theorem 1. In the next paragraph, we explain the construction of an algebraic model for Γ_G suitable for computations. In § 6, we indicate briefly why this is a model for Γ_G .

5. CONSTRUCTION OF AN ALGEBRAIC MODEL FOR THE SPACE OF SECTIONS OF A FIBER BUNDLE ([20], [18], [13]).

As a guide, consider first the geometric situation. Let $p: E \rightarrow M$ be a fiber bundle with base space M , fiber F and let Γ be the space of continuous sections of E .

We have the commutative diagramm

$$\begin{array}{ccc}
 & & e \\
 & & \longrightarrow \\
 M \times \Gamma & \xrightarrow{\quad} & E \\
 \downarrow & \searrow & \swarrow p \\
 & & M \\
 \downarrow & & \downarrow \\
 \Gamma & & * \\
 & \searrow & \\
 & & *
 \end{array}$$

1)

where e is the evaluation map associating to the point x of M and the section s the point $s(x)$ of E . The other maps are natural projections (* is a point).