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is the free commutative algebra $\Lambda(s, e)$, where $\deg s = 1$ and $ds = e$. As a B-module, it is free with basis 1 and s. A model for E_G is just $A \otimes \Lambda(x_\alpha)$.

As model for Γ_G, we take $R[e] \otimes \Lambda(x_\alpha, \bar{x}_\alpha)$, where $\deg \bar{x}_\alpha = \deg x_\alpha - 1$, the image of x_α by ε being $1 \otimes x_\alpha + s \otimes \bar{x}_\alpha$. The differential d is described as follows (compare with Sullivan [18] or [19]). Let h be the derivation of degree -1 of $\Lambda(x_\alpha, \bar{x}_\alpha)$ given by $hx_\alpha = \bar{x}_\alpha$ and $h\bar{x}_\alpha = 0$. Then if d_0 denotes the differential in $\Lambda(x_\alpha)$ identified to a subalgebra of $\Lambda(x_\alpha, \bar{x}_\alpha)$, we have

$$de = 0, dx_\alpha = d_0 x_\alpha - e\,\bar{x}_\alpha, d\bar{x}_\alpha = -hd_0 x_\alpha$$

Remark. In the case where E is the bundle described in § 4, its minimal model $A \otimes \Lambda(x_\alpha)$ over M_G is complicated, because there is an infinite number of generators x_α (except for $n=1$) labelled by a basis of the rational homotopy of a wedge of spheres, so by a basis of the free graded Lie algebra $L(n)$ generated by the spheres of this wedge (cf. [13]).

6. Sketch of the proof of the main theorem and applications

We represent the universal principal G-bundle as a limit of finite dimensional bundles P_k and we denote by Ω_P the inverse limit of algebras of forms Ω_{P_k}.

First note that we can replace $C^*(L_M; G)$ by the DG-algebra $C^*(L_M, \Omega_P)_G$ of G-basic elements in $C^*(L_M, \Omega_P)$ (compare with Cartan [5], exposé 20).

A model for E_G will be the algebra $C^*_\Delta(L_M, \Omega_{M \times P})_G = [C^*_\Delta(L_M, \Omega_M \hat{\otimes} \Omega_P]_G$ and a model for the evaluation map will be the inclusion of this DG-algebra in $C^*(L_M, \Omega_{M \times P})_G$.

In the construction of § 5, we choose $B = \Omega_{BG}$ as model for BG and, instead of taking for A a finite dimensional module over B, we take the DG-algebra $\Omega_{M_G} \approx [\Omega_{M \times P}]_G$ as model for M_G. We have to build the model for Γ_G along the same lines as in § 5, but in more intrinsic terms like in [13]. The minimal model (or Postnikov decomposition of E_G) will be of the form $A \otimes S^*(V)$, where $S^*(V)$ denotes the algebra of symmetric multilinear forms on a graded vector space V (cf. [13]).

As an algebra, the model for Γ_G will be the algebra $S^*_B(A \otimes V, B)$ of continuous symmetric B-multilinear forms on the graded B-module $A \otimes V$. One can construct a map of this model in $C^*(L_M, \Omega_{M \times P})_G$ and prove that it induces an isomorphism in cohomology.

Similarly, one can prove that $S_B^*(A \otimes V, B)$ is effectively a model for the space of sections Γ_G (cf. [14]).

Eventually for computations, one proves that one gets also a model for Γ_G by using instead of Ω_{M_G} a DG-algebra A as in § 5 which is a finite dimensional free B-module.

7. Example of a computation

Let us consider the case where M is the n-sphere S^n, G the rotation group SO_{n+1} and E the bundle described in § 3. A model for M_G is the DG-algebra A defined by

$$A = R[p_1, \ldots, p_k, s] / (s^2 - p_k) \quad d \equiv 0 \quad \text{for} \quad n = 2k$$

$$\text{or} \quad A = R[p_1, \ldots, p_{k-1}, \chi] \otimes E(s) \quad ds = \chi \quad \text{for} \quad n = 2k-1$$

where $\deg p_i = 4i$ and $\deg s = n$.

A model for E_G is obtained by taking the tensor product of A with WU_n, the differential being defined by

$$dh_i = c_i - p_{i/2} \quad \text{and} \quad dc_i = 0 .$$

By the way, WSO_n is also a model for E_G.

We now consider the case $n = 2$. The minimal model of E_G is the DG-algebra which begins as

$$A \otimes \Lambda(x_1, x_2, x_3, x_4, x_5, x_{12}, x_{13}, x_{23}, \ldots)$$

where

$$\deg x_1 = \deg x_2 = 5, \deg x_3 = 7, \deg x_4 = \deg x_5 = 8 ,$$
$$\deg x_{12} = 9, \deg x_{13} = \deg x_{23} = 11 ,$$

etc.

(there is an infinite number of generators).

The differential is defined by

$$dx_1 = dx_2 = 0, dx_3 = - p_l^2, dx_4 = p_1 x_1, dx_5 = p_1 x_2 ,$$
$$dx_{12} = x_1 x_2, dx_{13} = x_1 x_3 - p_1 x_4, dx_{23} = x_2 x_3 - p_1 x_5 ,$$

etc.

According to the construction of § 5, a minimal model for the bundle $\Gamma_G \to B_G$ begins as

$$R[p_1] \otimes \Lambda(\bar{x}_1, \bar{x}_2, x_1, x_2, \bar{x}_3, \bar{x}_4, \bar{x}_5, x_3, \bar{x}_{12}, x_4, x_5, \ldots)$$