Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 24 (1978)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON THE GELFAND-FUKS COHOMOLOGY

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Bibliographie

DOI: https://doi.org/10.5169/seals-49696

For that case, results similar to the one mentionned in this report have been obtained by Fuks-Segal (unpublished) and by T. Tsujishita [21].

For instance, when A is the algebra of smooth functions on M on which L_M acts by Lie derivative, their result is as follows. As it is described in § 3, the bundle E over M has a fiber F_n which is itself a principal U_n-bundle. Let us fix a fiber $F_n^{\circ} \approx U_n$ of this bundle; as it is invariant by the structural group $O_n \subset U_n$ of E, we get a subbundle E_o of E with typical fiber F_n^o. Then $C^* (L_M, A)$ will be a model for the inverse image of E_o by the evaluation map $M \times \Gamma \to E$.

2. One of the most interesting problems is to know when, for a given class α in $H^* (L_M)$, there is a space X and a foliation F on $X \times M$ transverse to the fibers such that the image of α in $H^* (X)$ by the characteristic homomorphism (cf. 2) is non zero.

Very recent and remarkable results of Fuchs [23] show that this is the case for all classes coming from WSO_n. (For earlier partial results, see [4].) One might expect that his method will apply in general and show that the answer is affirmative for all classes in $H^* (L_M)$ (and also for the similar problem with $H^* (L_M; G)$).

There is also the problem of the possible continuous variations of characteristic classes for flat bundles which would be interesting to study (cf. [23]).

REFERENCES

[1] Bott, R. A topological obstruction for integrability. *Proc. Symp. Pure Math. 16* (1970), pp. 127-131.

[2] —— On the Gelfand-Fuks cohomology, Differential Geometry. *Proc. of Symp. A.M.S. 27*, Part I, pp. 357-364.

[3] —— Lectures on characteristic classes and foliations (Notes by Lawrence Conlon). *Lecture Notes in Mathematics 279*, pp. 1-94, Springer Verlag, New York, 1972.

[4] —— On the characteristic classes of groups of diffeomorphisms. *Ens. Math. 23* (1977), pp. 209-220.

[5] Cartan, H. Exposés 19 et 20 du Séminaire Henri Cartan 1949-1950 (Espaces fibrés et homotopie).

[6] Gelfand, I. M. and D. Fuks. The cohomology of the Lie algebra of vector fields on the circle. *Funct. Anal. 3* (1968), pp. 92-93.

[7] —— The cohomology of the Lie Algebra of Tangent Vector fields on a Smooth Manifold I and II. *Funct. Anal. 3* (1969), pp. 32-52 and *4* (1970), pp. 23-32.

[8] —— The Cohomology of the Lie Algebra of Formal Vector Fields. *Izvestia Ak. SSSR 34* (1970), pp. 322-337.

[9] Godbillon, G. Cohomologie d'algèbres de Lie de champs de vecteurs formels. *Séminaire Bourbaki 421* (1972-1973).

[10] GUILLEMIN, V. Cohomology of vector fields on a Manifold. *Ad. in Math. 10* (1973), pp. 192-220.

[11] KOSZUL, J. L. Cohomologie des actions locales de groupes de Lie. *Symposia Matematica 16* (1975), pp. 399-407.

[12] HAEFLIGER, A. Sur les classes caractéristiques des feuilletages. *Séminaire Bourbaki 412* (1971-1972).

[13] —— Sur la cohomologie de l'algèbre de Lie des champs de vecteurs. *Ann. Scient. Ecole normale Supérieure, 4e série, 9* (1976), pp. 503-532.

[14] —— Un modèle pour l'espace des sections d'un fibré. *Séminaire sur les formes différentielles sur les ensembles simpliciaux*, 3e cycle romand de mathématiques (1974-1975) — Exposé No 3.

[15] —— Differentiable cohomology. Cours donné au CIME en 1976 *(to appear)*.

[16] HOCHSCHILD, G. and J. P. SERRE. Cohomology of Lie algebras. *Annals of Math. 57* (1963), pp. 591-603.

[17] LOSIK, M. V. Cohomology of the Lie algebra of Vector fields with coefficients in a Trivial Unitary Representation. *Func. Anal. 6* (1972), pp. 24-36.

[18] SULLIVAN, D. Infinitesimal Computations in Topology *(to appear)*.

[19] SULLIVAN, D. and M. VIGUÉ-POIRRIER. The homology theory of the closed geodesic problem. *J. of diff. geometry 11* (1976), pp. 633-644.

[20] THOM, R. L'homologie des espaces fonctionnels. *Colloque de Topologie algébrique*, Louvain 1956, pp. 29-39.

[21] TSUJISHITA, T. Continuous cohomology of the Lie algebra of vector fields. *Proc. Japan Acad. Ser. A* (1977), pp. 134-138.

[22] COHEN, F. R. and L. R. TAYLOR. Computations of Gelfand-Fuks cohomology, the cohomology of function spaces and the cohomology of configuration space *(to appear)*.

[23] FUCHS, D. Non-trivialité des classes caractéristiques de g-structures. Applications aux classes caractéristiques de feuilletages. *C. R. Acad. Sc. Paris, Série A, 2.4* (1977), pp. 1017-1019 et 1105-1108.

(Reçu le 2 décembre 1977)

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