THEOREM 3. *If* Ω *has a smooth real-analytic boundary, then* Ω *is regular.*

The essential ingredients in the proofs of these two theorems are contained in the next two [4].

THEOREM 4. *Let* Ω *be a pseudo-convex domain with a smooth boundary, and let* $a \in \partial\Omega$. *Let* U *be a neighbourhood of* a *in* $\mathbf{C}^n$ *such that* $\partial\Omega \cap U$ *is real-analytic.*

Let $q > 0$, *and suppose that* $\partial\Omega \cap U$ *contains the germ of a real-analytic set whose holomorphic dimension is* $\geqslant q$. *Then* $\partial\Omega \cap U$ *contains the germ of a* complex *analytic set of dimension* $\geqslant q$.

THEOREM 5. *Let* X *be a compact, real-analytic set in* $\mathbf{C}^n$. *Then* X *does not contain the germ of any complex analytic set of dimension* > 0.

Putting these results together, one obtains the following theorem.

THEOREM 6. *Let* Ω *be a bounded pseudo-convex domain in* $\mathbf{C}^n$ *with a smooth, real-analytic boundary. Then*

a) $\overline{\Omega}$ *has a fundamental system of neighbourhoods that are pseudo-convex, hence Stein.*

b) *For any* $a \in \partial\Omega$, *and any* $q > 0$, *the* $\bar{\partial}$*-Neumann problem is subelliptic at* a *for forms of type* (p, q).

These results and techniques are being very actively pursued at present. Many problems which looked inaccessible until recently have been solved, at least in important special cases. For instance, the Mergelyan theorem for $\overline{\Omega}$ has seen significant progress (see e.g. [9]). So has the question of global defining equations for the boundary of a pseudo-convex domain ([3]). Finally, a beginning has been made in the study of domains whose boundaries do contain complex analytic sets of positive dimension ([2]).

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