we conclude that $d(\{S_h\}, U(H)-T(H)) \geqslant 2 - \sigma_1$. Consequently, by Lemma 5.2,

$$\sigma_3 \geqslant 2 - \sigma_1 \,. \tag{5.2}$$

In order to prove that this inequality is sharp, we consider the point S_w, where w is the restriction to H of a branch of the logarithm. Since the boundary of $w(H)$ is not a quasicircle, $S_w \in U(H) - T(H)$. From $S_w(z) = z^{-2}/2$ it follows that $\| S_w \|_H = 2$. Let h be determined by the condition $S_h = r\, S_w$, $0 < r < 1$, and set $A = h(H)$. From $\| S_h \|_H < 2$ it follows that $S_h \in T(H)$, and so ∂A is a quasicircle. Now

$$\sigma_3 = d(\{S_h\}, U(H)-T(H)) = \| S_w - S_h \| = 2(1-r) = 2 - \sigma_1 \,,$$

showing that (5.2) is sharp.

Suppose that $2 \leqslant \sigma_1 < 6$. We then conclude from the remark at the end of 5.3 that, even though $\sigma_3 > 0$ for each A, we have $\inf \sigma_3 = 0$ for every σ_1.

Similarly, Lemma 5.2 can be used to deriving the upper estimate

$$\sigma_3 \leqslant \min(2, 6 - \sigma_1) \,.$$

(For the details we refer to [9].)

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