in case of n odd; the maximal root $\rho = -\delta_{l-1} - \delta_l$ is real, $\overline{\rho\,(S^{-1}\,\overline{H}\,S)} = \rho\,(H)$. Hence, the contact structure on $T\,G_0'\,T^{-1}/T\,P_0'\,T^{-1}$ is that obtained from G/P by 2.11. We conclude:

G_0/P_0 and $T\,G_0'\,T^{-1}/T\,P_0'\,T^{-1}$, the latter isomorphic to G_0'/P_0', are two reals forms of the complex contact manifold G/P.

5.6. We observed in 5.3 that the space of co-directions in complex projective space P^3, by means of Plücker's line geometry, is isomorphic to the space of lines in the quadric Ω^4 in complex P^5, and that this isomorphism makes real line geometry correspond to a real form of Ω^4. We found in 5.4 and 5.5 that the space of oriented co-directions in complex Euclidean space E^3 of Lie's higher sphere geometry, which is the space of lines in the quadric Ψ^4 in complex P^5, is isomorphic to the space of lines in the quadric Ω^4 also, and that this isomorphism makes real sphere geometry correspond to a second real form of Ω^4. That is, real line geometry and real sphere geometry are two distinct real forms of complex line geometry. The line-sphere transformation establishes the isomorphism of the spaces of lines in Ψ^4 and lines in Ω^4. The former places real sphere geometry in the foreground, the latter, real line geometry.

5.7. The isomorphism of 5.3 may be used to describe sphere geometry in terms of co-directions in complex P^3. Real sphere geometry then leads to the real form $PSU\,(2,2)$ of $PSL\,(4;\,\mathbf{C})$.

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