4. Absolutely isolated double points

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functions f listed in column 1 of Table 1 have minimal resolutions as in column 3. (I believe that this first appeared in [Hirzebruch 1].) The converse follows since the singularities listed are taut [Brieskorn 2; Tjurina 3; Laufer 4]. (Two resolutions $\pi: M \to V$ and $\pi': M' \to V'$ are *topologically equivalent* if their exceptional sets are homeomorphic by a homeomorphism preserving the self-intersection numbers. A singularity V is *taut* if any other singularity with a good resolution topologically equivalent to a good resolution of V is then isomorphic to V.)

The classification of rational double points has been generalized in several ways: to rational triple points [Artin, p. 135], to elliptic singularities [Wagreich 1], and to minimally elliptic singularities [Laufer 5]. The Dynkin diagrams B_n, C_n, F_4 and G_2 occur when resolving singularities over non-algebraically closed fields [Lipman 1]. There is also a relation with simple complex Lie groups [Brieskorn 3].

4. Absolutely isolated double points

There are at least three methods of resolving the singularity of the germ of a normal two-dimensional complex space V. The first method is one of local uniformization; this is originally due to Jung, and is described in detail in [Laufer 1]. The second method, due to Zariski, is to alternately blow up points and normalize. The third method (which generalizes to higher dimensions), is to blow up points and non-singular curves.

The singularity of V is *absolutely isolated* if it may be resolved by blowing up points alone, that is, it is not necessary to normalize or blow up curves. For example, the singularity of the zero locus of $f(x, y, z) = x^k + y^k + z^k$ is absolutely isolated, since it may be resolved by blowing up the origin once.

The singularity of V is a *double point* if its local ring is of multiplicity two. If V is $f^{-1}(0)$, this is equivalent to the lowest non-zero homogeneous term in the power series expansion of f being quadratic.

Characterization A4. The singularity of $f^{-1}(0)$ is an absolutely isolated double point.

The equivalence of Characterizations A1 and A4 was proved directly in [Kirby]. Later, it was shown [Tjurina 2; Lipman 1] that all rational singularities are absolutely isolated (thus showing Characterization A2 implies A4), and in [Brieskorn 1, Satz 1] that A4 implies A3.