# 4. CONCLUDING REMARKS

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of $\mathscr{E}$ formed by those germs $\tilde{f}$ which, in addition to the above conditions, satisfy $f'_w(z_0, w_0) \neq 0$. The classical existence and uniqueness theorem for implicit function equations allows us to associate to the germ $\tilde{f} \in \mathscr{U}$ of $f \in \mathscr{H}(U; \mathbf{C})$ at $(z_0, w_0)$ satisfying $f(z_0, w_0) = 0$, $f'_w(z_0, w_0) \neq 0$, the germ $\tilde{g} \in \mathscr{H}(z_0, \mathbf{C})$ of $g \in \mathscr{H}\left(B_\delta(z_0); \mathbf{C}\right)$ at $z_0$. It can be proved that the mapping $\tilde{f} \in \mathscr{U} \mapsto \tilde{g} \in \mathscr{H}(z_0; \mathbf{C})$ is holomorphic. We may repeat here some comments which are analogous to those made at the end of the above Example 3.

## 4. Concluding remarks

This article was written to attract prospective users in applications of holomorphy in infinite dimensions.

I have tried to illustrate through four very simple, classical examples, how the concept of holomorphic mappings in infinite dimensions comes up naturally in Analysis. The difference between Examples 1 and 2 on one side, and Examples 3 and 4 on the other side is striking: The first two examples seem very straightforward, while the last two examples look more sophisticated. However, sophistication in Mathematics is a matter of lack of habit; I personally am by now so used to dealing with germs of holomorphic functions that I no longer think of the last two examples as being sophisticated at all. Moreover, dealing long enough with any mathematical concept, particularly in applying it, leads to the development of a sort of intuition in that respect.

In 1963, I had my first opportunity of visiting Warsaw, and of talking leisurely to Mazur. I then played a little bit the role of a newspaper reporter and asked him if he, Banach and other members of the Polish group that developed Banach space theory, had specific applications in mind. Mazur answered, without any surprise to me as a mathematician, that the Polish group was guided by a conscience of the importance of Banach spaces in Mathematics proper. We witness nowadays how Banach spaces methods and results spread out in Mathematics and its applications. More accurately, Banach spaces have even been superseded by locally convex spaces for many of such goals. Psychologically, it is interesting to notice that the concept of a Banach space was also emphasized by Norbert Wiener; however Banach had a guiding idea to develop a fruitful theory, but that does not seen to be the case of Wiener in this particular instance. Likewise, the group that is developing holomorphy in infinite dimensions has been guided by a feeling of its possible interest in Mathematics proper. A promising direction of research at present seems to be the study of holomorphy linked to nuclearity in the sense of Grothendieck; interesting results in this direction have already been obtained, mainly by Boland and Dineen, but many such useful methods are to be expected in this area. Holomorphy in infinite dimensions is being used in Mathematical Physics, say in studying Fock spaces; and in Electrical Engineering through ideas originated from Volterra. However, the ties between the existing theory, or the theory to be developed, and its possible applications, are still loose, the reason being a lack of suitable interplay between mathematicians and users.

## 5. Some bibliographical references

The following references consist exclusively of some expository texts, and the proceedings of meetings. The readers should be able to trace back further information through them, concerning the various directions in which holomorphy in infinite dimensions branched off and is used. May I cite Kiselman in [16] below. He describes a problem in finite dimensions which was one of his motivations for the use of holomorphy in infinite dimensions; the problem has to do with the determination of the polynomially convex porters of a continuous linear form on $\mathscr{H}(\mathbf{C}^n; \mathbf{C})$ from the knowledge of its nonlinear Fourier-Borel transform. Actually, when I told Kiselman that I was preparing an article of motivation like the present one, he gladly wrote his article in [16] below, and suggested that the complete title of my article should be "Why Holomorphy in Infinite Dimensions? Why not?" According to an oral communication that I got from Dieudonné, one of the first authors to deal with holomorphy in infinite dimensions was D. Hilbert, in his article "Wesen und Ziele einer Analysis der unendlichvielen unabhägigen Variablen", *Rendiconti del Circolo Matematico di Palermo 27*, 1909, 59-74, or *Gesammelte Abhandlungen III*, 56-72.