Logic and quantum mechanics

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both a question of more exact answers for the case 2 and a more general question of extending the study from 2 to greater n. In fact C. G. Jockush generalizes and settles both questions shortly afterwards *(Journal of symbolic logic*, vol. 37, 1972, pp. 268-280): for every $n \geqslant 2$ and every recursive partition of n-elements sets of natural numbers, there is some Π_n set of indiscernibles; for every $n \geqslant 2$, there is some recursive partition of n-elements sets (into two classes) such that there is no Σ_n set of indiscernibles. In 1977, results by Kirby and Paris aroused widespread interest because their work yields some more mathematical examples of the incompleteness of Peano arithmetic (see, e.g., the last chapter of *Handbook of mathematical logic*, December 1977). At this juncture, several people observed that Jockusch's generalization of Specker's result actually yields quite directly rather similar results, one form of which says simply that Ramsey's theorem is undecidable in the weak (or predicative) second order extension of Peano arithmetic. (A version of the derivation is reported in my *Popular lectures on mathematical logic* being published in Beijing.)

This is in my opinion an illustration of how a good choice of an apparently isolated concrete problem can relate to more substantial developments in a surprising way. Pursuing this line of thought, I would also like to consider another seemingly small beginning by Specker which has been followed by larger results and new vistas.

LOGIC AND QUANTUM MECHANICS

In 1960, Specker began his consideration of propositions which are not simultaneously decidable with an ancient story about applying to marry a certain princess. Three boxes A, B, C are each empty or contain a ball. The problem is to guess which box is empty and which is not by selecting to open any two boxes which both are conjectured to be empty or non-empty. The boxes are connected in such a way that one can open any two boxes but then the third can no longer be opened. Moreover, the construction is such that whenever any two boxes are open, exactly one of them is empty. Hence nobody was able to win. Finally, somebody insisted on opening two boxes exactly one of which he conjectured to be empty. As he happened to guess right and the third box could no longer be opened, he won the hand of the princess. In this example, the three propositions "x is empty" ($x=A$, B, C) are not simultaneously decidable.

Specker then offers an improved formulation of the logic of quantum mechanics first considered by G. Birkhoff and J. v. Neumann (*Annales of Math.*, 1936). The logic is "partial" because the conjunction (say) of two propositions not simultaneously decidable has no truth value. He proposes, in contrast to Birkhoff and v. Neumann, that conjunctions, etc. of propositions which are not simultaneously decidable can also be introduced. The problem is then posed: "Can the description of a quantum-mechanical system be so extended with auxiliary (fictitious) propositions that the classical propositional logic holds?" This is answered in the negative by showing that the partial Boolean algebra $B(E^3)$ of the linear subspaces of the three dimensional Hilbert space cannot be imbedded in a Boolean algebra.

These observations in Specker's paper of 1960 were followed by systematic joint work with Kochen. The task of axiomatizing the logic of quantum mechanics is accomplished in 1965*a* and 1965*b* with a calculus of partial propositional functions that is shown to satisfy the natural requirements. They continued with the larger paper 1967*a* in which interesting examples are given to show that certain simple partial Boolean subalgebras of $B(E^3)$ cannot be imbedded in a Boolean (commutative) algebra. This yields an improvement (and correction) of J. v. Neumann's well-known theorem on the non-existence of hidden variables in quantum mechanics (*Mathematical foundations of quantum mechanics*, in German, 1932; English translation, 1955). For example, the result is summarized in E. J. Belinfante's *A survey of hidden variables* under the name "the Kochen-Specker paradox" (see p. 37).

Very recently, Kochen is circulating a typescript entitled *The interpretation of quantum mechanics* (89 pp) in which the negative results with Specker are turned around and expanded in all directions to get a new interpretation of quantum mechanics. The concept of interactive properties is taken seriously and elucidated mathematically. It will be interesting to watch how this will be received by physicists and mathematicians specializing in quantum theory.

I should like to turn to an area outside the mainstream of current logic in which Specker's work occupies a central place. This is the area of trying to strengthen the simple theory of types without going into the transfinite types. The best known proposal is Quine's New Foundations (briefly NF, *Am. math. monthly*, vol. 44, 1937, pp. 70-80) which strikes one as highly artificial. Specker not only proves most surprising results about NF but also gives a much more natural equivalent characterization of NF in terms of typical ambiguity.