

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 28 (1982)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FROBENIUS RECIPROCITY AND LIE GROUP REPRESENTATIONS ON $\bar{\Delta}$ COHOMOLOGY SPACES
Autor: Williams, Floyd L.

Bibliographie
DOI: <https://doi.org/10.5169/seals-52231>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 23.01.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

5. It is possible to formulate Frobenius reciprocity for unitary representations on a Hilbert space $\mathcal{H}(D)$ of L_2 -solutions of an invariant elliptic differential operator D on homogeneous bundles over a homogeneous space G/H whose isotropy subgroup H is compact modulo the center of G . Here G is a connected unimodular Lie group (not necessarily semisimple) subject to some mild structural constraints. In [33] Connes and Moscovici show that $\mathcal{H}(D)$ decomposes as a finite direct sum of irreducible unitary representations all of which are square-integrable modulo the center of G and occur with finite multiplicity. They derive for $\mathcal{H}(D)$ a reciprocity analogous to that expressed for the L_2 -cohomology spaces in Theorem 3.15 and Theorem 4.3.

REFERENCES

- [1] AHIEZER, D. Cohomology of compact homogeneous spaces. *Mat. Sb.* 84 (126) (1971), 290-300 = *Math. USSR Sb.* 13 (1971), 285-296.
- [2] ANDREOTTI, A. and E. VESSENTINI. Carleman estimates for the Laplace-Beltrami operator on complex manifolds. *Publ. I.H.E.S.* No. 25 (1965), 313-362.
- [3] ATIYAH, M. Elliptic operators, discrete groups and von Neumann algebras. *Astérisque* 32-33 (1976), 43-72.
- [4] ——— *Characters of semisimple Lie groups*. Mimeographed lecture notes, Univ. of Oxford.
- [5] ——— The Harish-Chandra character. *Lecture No. 7 from Proceedings of the SRC/LMS Research Symposium on Representations of Lie Groups, London Math. Soc. Series 34*, Cambridge Univ. Press, 176-181.
- [6] ATIYAH, M. and W. SCHMID. A geometric construction of the discrete series for semisimple Lie groups. *Inventiones Math.* 42 (1977), 1-62.
- [7] ——— A new proof of the regularity theorem for invariant eigendistributions on semisimple Lie groups. *To appear*.
- [8] AUSLANDER, L. and B. KOSTANT. Polarization and unitary representations of solvable Lie groups. *Inventiones Math.* 14 (1971), 255-354.
- [9] BERNAT, P., N. CONZE, M. DUFLO, M. LÉVY-NAHAS, M. RAIS, P. RENOARD et M. VERGNE. *Représentations des groupes de Lie résolubles*. Monographies de la Société Mathématique de France, Dunod, éditeur. (1972) Paris.
- [10] BLATTNER, R. On induced representations. *Amer. J. of Math.* 83 (1961), 79-98.
- [11] BOREL, A. et A. WEIL. Représentations linéaires et espaces homogènes Kähleriens des groupes de Lie compacts. *Séminaire Bourbaki*, May 1954 (exposé by J.-P. Serre).
- [12] BOTT, R. Homogeneous vector bundles. *Annals of Math.* (2) 66 (1957), 203-248.
- [13] ——— Induced representations. *Seminars on Analytic Functions, Vol. 2*, Institute for Advanced Study, Princeton, N.J., 1957.
- [14] BRUHAT, F. Travaux de Harish-Chandra. *Séminaire Bourbaki*, exposé 143 (1957), 1-9.
- [15] CARMONA, J. Représentations du groupe de Heisenberg dans les espaces de $(0, q)$ -formes. *Math. Ann.* 205 (1973), 89-112.
- [16] CARTAN, E. Les tenseurs irréductibles et les groupes simples et semisimples. *Bull. Sci. Math.* 49 (1925), 130-152.

- [17] CASSELMAN, W. and M. OSBORNE. The n -cohomology of representations with an infinitesimal character. *Compositio Math.* 31 (1975), 219-227.
- [18] HARISH-CHANDRA. On some applications of the universal enveloping algebra of a semisimple Lie algebra. *Trans. Amer. Soc.* 70 (1951), 28-96.
- [19] ——— Representations of a semisimple Lie group on a Banach Space I. *Trans. Amer. Math. Soc.* 75 (1953), 185-243.
- [20] ——— Representations of semisimple Lie groups II, III. *Trans. Amer. Math. Soc.* 76 (1954), 26-65 and 234-253, respectively.
- [21] ——— Representations of semisimple Lie groups IV, V. *Amer. J. of Math.* 77 (1955), 743-777 and 78 (1956), 1-41, respectively.
- [22] ——— Representations of semisimple Lie groups VI: Integrable and square-integrable representations. *Amer. J. of Math.* 78 (1956), 564-628.
- [23] ——— The characters of semisimple Lie groups. *Trans. Amer. Math. Soc.* 83 (1956), 98-163.
- [24] ——— Invariant eigendistributions on a semisimple Lie group. *Trans. Amer. Math. Soc.* 119 (1965), 457-508.
- [25] ——— Discrete series for semisimple Lie groups I. *Acta Math.* 113 (1965), 241-318.
- [26] ——— Discrete series for semisimple Lie groups II: Explicit determination of the characters. *Acta Math.* 116 (1966), 1-111.
- [27] ——— Two theorems on semisimple Lie groups. *Annals of Math.* (2) 83 (1966), 74-128.
- [28] ——— Harmonic analysis on semisimple Lie groups. *Bull. Amer. Math. Soc.* 76 (1970), 529-551.
- [29] CHEVALLEY, C. *Theory of Lie groups I*. Princeton Math. Series, No. 8, Princeton Univ. Press, Princeton, N.J., 1946.
- [30] CHEVALLEY, C. and S. EILENBERG. Cohomology theory of Lie groups and Lie algebras. *Trans. Amer. Soc.* 63 (1948), 85-124.
- [31] CONNES, A. and H. MOSCOVICI. The L_2 -index theorem for homogeneous spaces. *Bull. Amer. Math. Soc.* 1 (1977), 688-690.
- [32] ——— The L_2 -index theorem for homogeneous spaces of Lie groups. *To appear*.
- [33] ——— Invariant elliptic equations and discrete series representations. *Preprint I.H.E.S.*, March 1980.
- [34] DEMAZURE, M. Une démonstration algébrique d'un théorème de Bott. *Inventiones Math.* 5 (1968), 349-356.
- [35] DOLBEAULT, P. Formes différentielles et cohomologie sur une variété analytique complexe I. *Annals of Math.* (2) 64 (1956), 83-130.
- [36] DUFLO, M. Représentations de carré intégrable des groupes semisimple réels. *Séminaire Bourbaki*, Vol. 1977/78, exposés 507-524, Springer-Verlag Lecture Notes No. 710, 22-40.
- [37] FROBENIUS, G. Über relationen zwischen den characteren einer gruppe and ihrer untergruppen. *Sitzungsb Kön Preusz. Akad. Wiss. zu Berlin* (1898), 501-515.
- [38] ——— Über die composition der caractere einen gruppe. *Sitzungsb Kön Preusz. Akad. Wiss. zu Berlin* (1899), 330-339.
- [39] GRIFFITHS, P. Some geometric and analytic properties of homogeneous complex manifolds Part I: Sheaves and cohomology. *Acta Math.* 110 (1963), 115-155.
- [40] GRIFFITHS, P. and W. SCHMID. Locally homogeneous complex manifolds. *Acta Math.* 123 (1969), 253-302.
- [41] GUNNING, R. and H. ROSSI. *Analytic functions of several complex variables*. Prentice-Hall, Englewood Cliffs, N.J., 1967.
- [42] HELGASON, S. *Differential geometry and symmetric spaces*. Academic Press, New York and London, 1962.

- [43] HIRZEBRUCH, F. *Topological methods in algebraic geometry*. Die Grundlehren der Math. Wissenschaften, Band 131, Springer-Verlag, Berlin, Heidelberg, New York, 1966.
- [44] HOCHSCHILD, G. and J.-P. SERRE. Cohomology of Lie algebras. *Annals of Math. (2)* 57 (1953), 591-603.
- [45] HOTTA, R. On realization of the discrete series for semisimple Lie groups. *J. Math. Soc. Japan* 23 (1971), 384-407.
- [46] HUMPHREYS, J. *Introduction to Lie algebras and representation theory*. Graduate Texts in Math. Vol. 9, Springer-Verlag, Berlin, Heidelberg, New York, 1973.
- [47] ISE, M. Some properties of complex analytic vector bundles over compact complex homogeneous spaces. *Osaka Math. J.* 12 (1960), 217-252.
- [48] JACOBSON, N. *Lie algebras*. Interscience Tracts in Pure and Appl. Math. No. 10, Interscience, New York, 1962.
- [49] KIRILLOV, A. Unitary representations of nilpotent Lie groups. *Uspekhi Mat. Nauk.* 17 (1962), 57-110 — Russian Math. Surveys 17 (1962), 53-104.
- [50] KOSTANT, B. Lie algebra cohomology and the generalized Borel-Weil theorem. *Annals of Math. (2)* 74 (1961), 329-387.
- [51] ——— Orbits, symplectic structures and representation theory. *Proc. U.S.-Japan Seminar Diff. Geom.*, Kyoto, Japan, 1965.
- [52] KOSTANT, B. M.I.T. Lecture Notes (*unpublished*), 1967.
- [53] ——— On certain representations which arise from a quantization theory. *Springer-Verlag Lecture Notes in Physics* 6 (1970), 237-253, Berlin, Heidelberg, New York.
- [54] LANGLANDS, R. Dimension of spaces of automorphic forms. *Proc. of Symposia in Pure Math. IX* (1966), 253-257.
- [55] MACKEY, G. Induced representations of locally compact groups I. *Annals of Math. (2)* 55 (1952), 101-139.
- [56] ——— Induced representations of locally compact groups II. *Annals of Math. (2)* 58 (1953), 193-221.
- [57] MOORE, C. and J. WOLF. Square integrable representations of nilpotent groups. *Trans. Amer. Math. Soc.* 185 (1973), 445-462.
- [58] MOSCOVICI, H. A vanishing theorem for L^2 -cohomology in the nilpotent case. *Non-commutative harmonic analysis*, Springer-Verlag Lecture Notes in Math. 728 (1979), 201-210, Berlin, Heidelberg, New York.
- [59] MOSCOVICI, H. and A. VERONA. Harmonically induced representations of nilpotent Lie groups. *Inventiones Math.* 48 (1978), 61-73.
- [60] NARASIMHAN, M. and K. OKAMOTO. An analogue of the Borel-Weil-Bott theorem for hermitian symmetric pairs of non-compact type. *Annals of Math. (3)* 91 (1970), 486-511.
- [61] OKAMOTO, K. On induced representations. *Osaka J. of Math.* 4 (1967), 85-94.
- [62] ——— On square-integrable $\bar{\partial}$ -cohomology spaces attached to homogeneous symplectic manifolds. Unpublished manuscript, Institute for Advanced Study, Princeton, N.J.
- [63] OKAMOTO, K. and H. OZEKI. On square-integrable $\bar{\partial}$ -cohomology spaces attached to hermitian symmetric spaces. *Osaka J. of Math.* 4 (1967), 95-110.
- [64] PARTHASARATHY, R. A note on the vanishing of certain L_2 -cohomologies. *J. of Math. Soc. Japan* 23 (1971), 676-691.
- [65] ——— Dirac operator and the discrete series. *Annals of Math.* 96 (1972), 1-30.
- [66] ——— An algebraic construction of a class of representations of a semisimple Lie algebra. *Math. Ann.* 226 (1977), 1-52.

- [67] PENNEY, R. Canonical objects in the Kirillov theory of nilpotent Lie groups. *Proc. Amer. Math. Soc.* 66 (1977), 175-178.
- [68] — Harmonically induced representations of nilpotent Lie groups and automorphic forms on nilmanifolds. *Trans. Amer. Math. Soc.* 260 (1980), 123-145.
- [69] — Lie cohomology of representations of nilpotent Lie groups and holomorphically induced representations. *Preprint*, 1979.
- [70] PETER, F. und H. WEYL. Die vollständigkeit der primitiven darstellungen einer geschlossenen kontinuierlichen gruppe. *Math. Ann.* 97 (1927), 727-755.
- [71] *Proceedings of Symposia in Pure Math., Harmonic analysis on homogeneous spaces*, vol. 26 (1973), A.M.S., Providence, R.I.
- [72] PUKANSZKY, L. *Leçons sur les représentations des groupes*. Monographies de la Société Mathématique de France, Dunod, éditeur (1967), Paris.
- [73] RAMANAN, S. Holomorphic vector bundles on homogeneous spaces. *Topology* 5 (1966), 159-177.
- [74] ROSENBERG, J. Realization of square-integrable representations of unimodular Lie groups on L^2 -cohomology spaces. *Trans. Amer. Math. Soc.* 261 (1980), 1-32.
- [75] SATAKE, I. Unitary representations of a semi-direct product of Lie groups on \bar{d} -cohomology spaces. *Math. Ann.* 190 (1971), 177-202.
- [76] SCHIFFMAN, G. Un analogue du théorème de Borel-Weil-Bott dans le cas non compact. *Séminaire Bourbaki*, June 1971, 323-336.
- [77] SCHMID, W. *Homogeneous complex manifolds and representations of semisimple Lie groups*. Ph.D. thesis, Univ. Calif., Berkeley, 1967.
- [78] — On a conjecture on Langlands. *Annals of Math.* 93 (1971), 1-42.
- [79] — Some properties of square-integrable representations of semisimple Lie groups. *Annals of Math.* 102 (1975), 535-564.
- [80] — On the characters of the discrete series. *Inventiones* 30 (1975), 47-144.
- [81] — Representations of semisimple Lie groups. *Lecture No. 8 from Proceedings of the SRC/LMS Research Symposium on Representations of Lie Groups, London Math. Soc. Series 34*, Cambridge Univ. Press, 185-235.
- [82] — L^2 -cohomology and the discrete series. *Annals of Math.* 103 (1976), 375-394.
- [83] — A geometric construction of the discrete series for semisimple Lie groups. *Lectures from the Nato Advanced Study Institute on Harmonic Analysis and Representations of Semisimple Lie Groups held at Liege, Belgium, Sept. 1977*, edited by M. Cahen, M. DeWilde, and J. Wolf; D. Reidel Publ. Company.
- [84] SINGER, I. Some remarks on operator theory and index theory, from K -theory and Operator Algebras. *Springer-Verlag Lecture Notes in Math.* 575 (1975), 128-138, Berlin, Heidelberg, New York.
- [85] VARADARAJAN, V. *Harmonic analysis on real reductive groups*. Springer-Verlag Lecture Notes in Math. 576 (1977), Berlin, Heidelberg, New York.
- [86] WAKIMOTO, M. Polarizations of certain homogeneous spaces and most continuous principal series. *Hiroshima Math. J.* 2 (1972), 483-533.
- [87] WALLACH, N. Induced representations of Lie algebras and a theorem of Borel-Weil. *Trans. Amer. Math. Soc.* 136 (1969), 181-187.
- [88] — *Harmonic analysis on homogeneous spaces*. Marcel Dekker, Inc., New York, 1973.
- [89] — Representations of semisimple Lie groups and Lie algebras. *Proceedings of the 1977 Seminar of the Canadian Math. Congress on Lie Theories and Their Applications, Queen's Papers in Pure and Applied Math. No. 48*, 154-246.
- [90] WANG, H. Closed manifolds with homogeneous complex structure. *Amer. J. of Math.* 76 (1954), 1-32.

- [91] WARNER, G. *Harmonic analysis on semisimple Lie groups, I, II*. 1972, Springer-Verlag, Vol. 188, 189, Berlin, Heidelberg, New York.
- [92] WELLS, R., Jr. *Differential analysis on complex manifolds*. Prentice-Hall, Englewood Cliffs, N.J., 1973.
- [93] WEYL, H. Theorie der darstellung kontinuierlichen halb-einfacher gruppen durch lineare transformationen. Teil I, *Mathematische Zeitschrift* 23 (1925), 271-309, Teil II, 24 (1926), 328-376, and Teil III, 24 (1926), 377-395.
- [94] ——— *The classical groups*. Princeton Univ. Press, Princeton, N.J., 1946.
- [95] WILLIAMS, F. Complex homogeneous bundles and finite-dimensional representation theory. *Proceedings of Symposia in Pure Math., Several complex variables, Vol. 30, Part 2* (1977), 317-320.
- [96] WOLF, J. The action of a real semisimple group on a complex flag manifold, I: Orbit structure and holomorphic arc components. *Bull. Amer. Math. Soc.* 75 (1969), 1121-1237.
- [97] ——— Complex manifolds and unitary representations. *Proceedings of the International Conference in Several Complex Variables*, Univ. Maryland, 1970, Springer-Verlag Lecture Notes in Math. 185 (1971), 242-287, Berlin, Heidelberg.
- [98] ——— Orbit method and nondegenerate series. *Hiroshima Math. J.* 4 (1974), 619-628.
- [99] ——— The action of a real semisimple group on a complex flag manifold, II: Unitary representations on partially holomorphic cohomology spaces. *Memoirs Amer. Math. Soc.*, No. 138, 1974.
- [100] ZELEBENKO, D. *Compact Lie groups and their representations*. « Nauka », Moscow, 1970; English transl. Transl. Math. Monographs, Vol. 40, Amer. Math. Soc., Providence, R.I., 1973.

(Reçu le 20 janvier 1981)

Floyd L. Williams

University of Massachusetts
Amherst, MA 01003
USA