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Artikel: FROBENIUS RECIPROCITY AND LIE GROUP REPRESENTATIONS ON $\bar{\delta}$ COHOMOLOGY SPACES

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5. It is possible to formulate Frobenius reciprocity for unitary representations on a Hilbert space $\mathcal{H}(D)$ of L_2-solutions of an invariant elliptic differential operator D on homogeneous bundles over a homogeneous space G/H whose isotropy subgroup H is compact modulo the center of G. Here G is a connected unimodular Lie group (not necessarily semisimple) subject to some mild structural constraints. In [33] Connes and Moscovici show that $\mathcal{H}(D)$ decomposes as a finite direct sum of irreducible unitary representations all of which are square-integrable modulo the center of G and occur with finite multiplicity. They derive for $\mathcal{H}(D)$ a reciprocity analogous to that expressed for the L_2-cohomology spaces in Theorem 3.15 and Theorem 4.3.

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