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# THE REPRESENTATION THEORY OF $SL(2, \mathbf{R})$ , A NON-INFINITESIMAL APPROACH

by Tom H. KOORNWINDER

## ABSTRACT

The representation theory of  $SL(2, \mathbf{R})$  is developed by the use of non-infinitesimal methods. This approach is based on an explicit knowledge of the matrix elements of the principal series with respect to the  $K$ -basis. The irreducible subquotient representations of the principal series are determined, and also their Naimark equivalences and unitarizability. All irreducible  $K$ -unitary,  $K$ -finite representations of  $SL(2, \mathbf{R})$  are classified, where an inversion formula for the generalized Abel transform provides an important tool.

## 1. INTRODUCTION

In 1947 two papers appeared on the representation theory of the two prototypes of noncompact semisimple Lie groups, namely by BARGMANN [2] on  $SL(2, \mathbf{R})$  and by GELFAND & NAIMARK [18] on  $SL(2, \mathbf{C})$ . The methods in the two papers are surprisingly different. Bargmann uses the infinitesimal (i.e. Lie algebraic) approach, while Gelfand & Naimark prefer non-infinitesimal (global) methods. In subsequent work to generalize these results for arbitrary noncompact semisimple Lie groups, the Bargmann approach has proved to be most successful, in particular by the work of Harish-Chandra. (However, it is interesting to note MAUTNER's [31] review of HARISH-CHANDRA's paper [22].)

Without denying the success of the infinitesimal approach, I want to add some motivation for a paper which favours the global approach:

- (a) *The didactic argument.* The global approach is a more natural and direct one and it does not require so much sophisticated functional analysis as the infinitesimal approach.