Zeitschrift: L'Enseignement Mathématique

Band: 28 (1982)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE REPRESENTATION THEORY OF SL(2, R), A NON-INFINITESIMAL APPROACH

Kapitel: 2.1. Preliminaries

Autor: Koornwinder, Tom H.

DOI: https://doi.org/10.5169/seals-52233

$SU(n, 1)$. For convenience, in order to avoid matrix manipulations, we restrict ourselves here to the case that the compact subgroup K is abelian.

The results of this paper may be generalized rather easily to the universal covering group of $SL(2, \mathbf{R})$. The extension to $SL(2, \mathbf{C})$ was done by KOSTERS [28], see also NAIMARK [34, ch. 3, §9]. Hopefully, an extension to $SO_0(n, 1)$ and $SU(n, 1)$ is feasible.

The reader of this paper is supposed to already have a modest knowledge about certain elements of semisimple Lie theory, like principal series and spherical functions. Suitable references will be given. Some of this preliminary material can also be found in the earlier version [27]. Modern accounts of the infinitesimal approach to $SL(2, \mathbf{R})$ can be found, for instance, in SCHMID [36, §2] or VAN DIJK [9]. TAKAHASHI [42] also presented a global approach to $SL(2, \mathbf{R})$, partly based on an earlier version of the present paper, partly (the global proof of Theorem 5.4) independently.

Finally, I would like to thank G. van Dijk and M. Flensted-Jensen for useful comments.

2. THE CANONICAL MATRIX ELEMENTS OF THE PRINCIPAL SERIES

2.1. PRELIMINARIES

Let G be a locally compact group satisfying the second axiom of countability (lcsc. group). A *Hilbert representation* of G is a strongly continuous but not necessarily unitary representation τ of G on some Hilbert space $\mathscr{H}(\tau)$ (which is always assumed to be separable). Let K be a compact subgroup of G. A Hilbert representation τ of G is called *K-unitary* if the restriction $\tau|_K$ of τ to K is a unitary representation of K. A Hilbert representation τ of G is called *K-finite* respectively *K-multiplicity free* if τ is K-unitary and each $\delta \in \hat{K}$ has finite multiplicity respectively multiplicity 1 or 0 in $\tau|_K$. If τ is K-multiplicity free then the *K-content* $\mathscr{M}(\tau)$ of τ is the set of all $\delta \in \hat{K}$ which have multiplicity 1 in $\tau|_K$.

Let K be a compact abelian subgroup of G and let τ be a K-multiplicity free representation of G. Choose an orthogonal basis $\{\phi_\delta \mid \delta \in \mathscr{M}(\tau)\}$ of $\mathscr{H}(\tau)$ such that

$$\tau(k)\phi_\delta = \delta(k)\phi_\delta, \qquad \delta \in \mathscr{M}(\tau), k \in K.$$

We call $\{\phi_\delta\}$ a *K-basis* for $\mathscr{H}(\tau)$ and the functions $\tau_{\gamma\delta}(\gamma, \delta \in \mathscr{M}(\tau))$, defined by

$$\tau_{\gamma,\delta}(g) := (\tau(g)\phi_\delta, \phi_\gamma), \quad g \in G, \tag{2.1}$$

the *canonical matrix elements* of τ (with respect to K).