2. Streets in all directions covering null area

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But also the triangle of Figure 2 (area: $1/\sqrt{3} = 0.5773502... < \pi/4$) is another such figure of smaller area.

For a long time many people thought that the problem would be solved by means of a sort of curvilinear triangle (see Fig. 3), a figure bounded by a hypocycloid γ with three cuspidal points inscribed in a circle of radius 3/4. This curve has the property that for each point M in γ the tangent at M to γ intersects γ in two other points A and B such that the length of AB is 1. The area of the figure enclosed by it is $\pi/8 = 0.392699...$ and it is easy to see that a needle of length 1 can turn around in any open set containing such a figure inside.

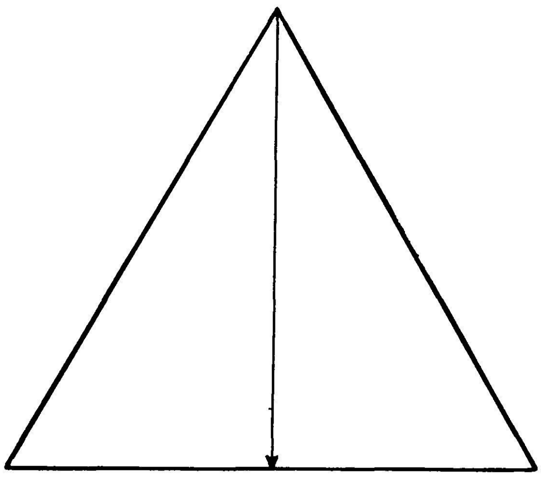

FIGURE 2

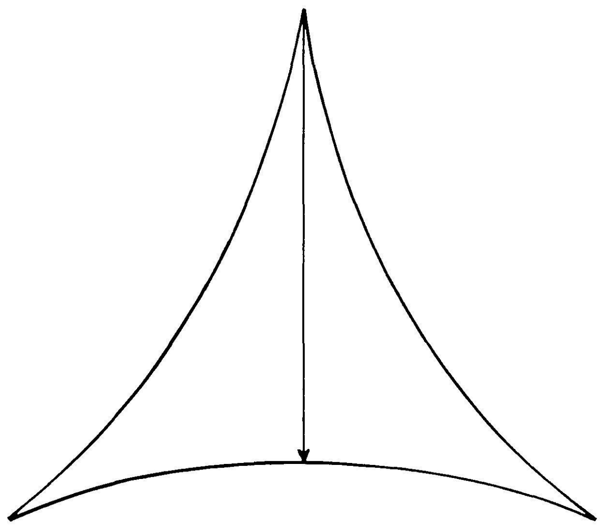

FIGURE 3

2. STREETS IN ALL DIRECTIONS COVERING NULL AREA

Kakeya's problem was in fact solved in 1919 by Besicovitch, but nobody, not even Besicovitch himself, realized it. He was at that time in Perm, in a University rather isolated from the rest of the mathematical world where the Kakeya problem did not arrive. On the other hand the tool created by Besicovitch for the solution of some other problem did not go very far from his place either. Besicovitch constructed a plane set of null area containing segments of length 1 in all directions. As we shall see this gives the following solution to the Kakeya problem: *Given any arbitrarily small* $\eta > 0$ *one can construct a plane figure with area smaller than* η *such that the needle can be continuously inverted inside it.*