Finally, it remains to verify that the equations $f(x, y, z) = 0$ in column (a′) have the resolutions of column (b′). This can be verified by blowing up points in three-space. (Note that by (4B⇔4A) above, the graphs are already known, and it is only necessary to match them with the equations.) The details are hard to write down, and will be omitted. This completes the proof of Theorem 2.

It would be interesting to understand the resolution of real singularities better. For example, what does the resolution of $x^3 + y^2 - z^2 = 0$ look like, topologically? It would also be interesting to understand the connections (if any) between the modality of a real germ as real germ and as complex germ.

More generally, the Dynkin diagrams B_k, C_k, and F_4 arise in situations where there is an involution on an object corresponding to the diagrams A_k, D_k and E_k. In the above theorem, the involution is conjugation. Connections with simple algebraic groups are discussed in [Slodowy, 6.2 and Appendix I]. Another example is critical points of functions on manifolds with boundary [Arnold 2]; these correspond to functions on the doubled manifold invariant with respect to the obvious involution. Lastly, the diagram G_2 arises where there is an automorphism of order 3 on an object corresponding to D_4.

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