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THE INFLUENCE OF COMPUTERS AND INFORMATICS ON MATHEMATICS AND ITS TEACHING

AN ICMI DISCUSSION DOCUMENT

prepared by

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Computers and informatics are changing all societies of our time. As the steam engine introduced the first industrial revolution, so the computer is introducing what is often called the second industrial revolution. That first revolution was accompanied by the development of the physical sciences; one must expect that new sciences connected with informatics will accompany the second. The prospects, then, are immense: new needs, new sciences, new technologies, new qualifications, the elimination of repetitive or laborious work, and, of course, new social challenges to be met.

Mathematics does not escape this movement, and this is why ICMI has taken the initiative of organising an international study on the theme: the influence of computers and informatics on mathematics and its teaching. As a first stage, the present document is being circulated for discussion. It is organised around three important questions:

1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science?

2. How can new curricula be designed to meet the needs and possibilities?

3. How can the use of computers help the teaching of mathematics?

So far as questions 2 and 3 are concerned, we are limiting our study to the curriculum and teaching at university and pre-university level (from the age of 16 years). School mathematics will be the subject of another ICMI international study. Naturally one will find the same topics and ideas appearing at every level, in answer to the questions posed. Two aspects, in particular, are essential: the influence of technology which allows better things to be done more quickly, and in different ways, and the influence of fundamental concepts of informatics, in the forefront of which is found algorithmics.

1. The effect on mathematics

Mathematical concepts have always depended on methods of calculation and methods of writing. Decimal numeration, the writing of symbols, the construction of tables of numerical values all preceded modern ideas of real number and of function. Mathematicians calculated integrals, and made use of the integration sign, long before the emergence of Riemann's or Lebesgue's concepts of the integral. In a similar manner, one can expect the new methods of calculation and of writing which computers and informatics offer to permit the emergence of new mathematical concepts. But, already today, they are pointing to the value of ideas and methods, old or new, which do not command a place in contemporary "traditional" mathematics. And they permit and invite us to take a new look at the most traditional ideas.

Let us consider different ideas of a real number. There is a point on the line R, and this representation can be effective for promoting the understanding of addition and multiplication. There is also an accumulation point of fractions, for example, continued fractions giving the best approximation of a real by rationals. There is also a non-terminating decimal expansion. There is also a number written in floating-point notation. Experience with even a simple pocket calculator can help validate the last three aspects. The algorithm of continued fractions—which is only that of Euclid—is again becoming a standard tool in many parts of mathematics. Complicated operations (exponentiation, summation of series, iterations) will, with the computer's aid, become easy. Yet even these simplified operations will give rise to new mathematical problems: for example, summing terms in two different orders (starting with the largest or starting from the smallest) will not always produce the same numerical result.

Again, consider the notion of function. Teaching distinguishes between, on the one hand, elementary and special functions—that is, those functions tabulated from the 17th to the 19th century—and, on the other, the general concept of function introduced by Dirichlet in 1830. Even today, to "solve" a differential equation is taken to mean reducing the solution to integrals, and if possible to elementary functions. However, what is involved in functional equations is the effective calculation and the qualitative study of solutions. The functions in which one is interested therefore are calculable functions and no longer only those which are tabulated. The theories of approximation and of the superposition of functions—developed well before computers—are now validated. The field of elementary functions is extended, and functions of a non-elementary nature are introduced naturally through the discretisation of non-linear problems. Informatics, too, compels us to take a new look at the notion of a variable, and at the link between symbol and value. This link is strongly exploited in mathematics (for example, in the symbolism of the calculus). In informatics, the necessity of working out, of realising the values has presented this problem in a new way. The symbolism of functions is not entirely transferable. This has resulted in computer languages of different types: thus the notion of a variable in LISP does not correspond exactly to that in some other languages in which variables have values.

For the last of our examples let us consider sets of points linked to dynamical systems, iteration of transformations or stochastic processes. The use of computers has brought new life to their study, both by physicists and mathematicians, and has given rise to a new terminology: for example, strange attractors, fractals.

From these examples it can be seen that computers and informatics have *stimulated* new research, *restored* to the mathematician's consideration questions recently neglected but previously studied over a long period of time, and *made possible* the study of new questions. We hope that as a result of the discussions connected with the ICMI study new light will be cast on each of these aspects.

There has always been an experimental side to mathematics. Euler insisted on the rôle of observation in pure mathematics: "the properties of numbers that we know have usually been discovered by observation, and discovered well before their validity has been confirmed by demonstration ... It is by observation that we increasingly discover new properties, which we next do our utmost to prove". Computers have suddenly greatly increased our possibilities for observation and experimentation in mathematics. The solution of the non-linear wave equation, the soliton, was discovered by numerical experimentation before it became a mathematical object, and gave rise to a rigorous theory. In the iteration of rational transformations it is the plots obtained by computers which have guided recent research. An entirely new art of experimentation is developing in all branches of mathematics. Calculations which were formerly impracticable are now easily accomplished; it is now a question of working out an appropriate plan of action. Visualisations are possible and they form a unifying bond between mathematicians in offering them subjects for study on which specialists from different disciplines can unite to work. There has been a considerable increase in the number and variety of stimuli which allow, indeed encourage, one to query and investigate their mathematical nature in order to establish and appreciate their inter-relationships. An awareness of these new possibilities has for some years penetrated research mathematics. Only on rare occasions has it been allowed to influence and infiltrate our teaching. However, these possibilities for experimentation, now practicable on a large scale, are most full of promise for the renewal and improvement of the teaching of mathematics.

Mathematics is also, and will remain, a science of proof. But the status of proof is not immutable. The level of rigour and degree of formalisation depends upon time and place. For some twenty years the fashion was for non-constructive proofs of existence theorems: methods of ideals for g.c.d., pigeon-hole principle for rational approximations, axiom of choice for functional analysis, probabilistic methods without explicit constructions, etc. Today the point of view has changed. Whenever possible, one makes use in a proof of an algorithm which permits one to obtain effectively the object sought.

Computers have had another effect on the status of proof, as has been shown in the celebrated case of the four-colour theorem. Until now, the most long and involved proofs were edited and published and the reader had access to and control over any exterior information required (tables, references). In principle, a mathematician working alone was supposed to be able to follow and verify every step of a proof thanks to this method of presentation. Now new types of proof have appeared: numerical proofs in which there occur numbers of a size and in quantities which preclude their being manipulated by hand, and algorithmic proofs dependent on the effectiveness and correctness of the algorithms. Computer-aided proofs have produced the need, therefore, for a new form of professional practice. This does not yet seem to have been codified. Doubtless it will be in the future.

Algorithms have played an important rôle in mathematics since Euclid, and even more since the birth of algebra. They constitute the most important mathematical constituent of informatics. We have referred to the rôle of algorithms as tools in proof and we are all aware that they are essential tools in calculation. Now, however, they are becoming more and more a study in themselves. Fascinating questions now arise on space and time complexity—on how to formulate algorithms so as to minimize computer storage space and running time—and on the development of algorithms suitable for processors running in parallel. To take but one example, by mathematical ingenuity the time complexity for the fast Fourier transform algorithm has been reduced from n^2 to $n \log n$, which is of considerable practical importance for large values of n. There are other problems concerned with the effectiveness of algorithms, their correctness and the way in which they can be elaborated. We note, as an example, the rôle of invariants and fixed points when establishing the correctness of algorithms.

One must also stress how algorithms are increasingly being called upon to play a central rôle in society: they arise in business and commerce, in technology and in automation. Mathematical problems arise then in many new domains, and mathematical methods have an increasingly far-reaching applicability.

Finally, from now on symbolic systems will enable the computer user to carry out difficult calculations within algebra and analysis. The possibilities raised are enormous, and one must take the measure of the actual performance of such systems and of their rôle in research mathematics, as well as of the influence they should have on the teaching of mathematics at the university and pre-university levels. Informatics, for example, extends the field of mathematical research on formal calculus.

2. The effect of computers on curricula

Curricula are generally the product of a long tradition, and their evolution is governed by two principal factors: the needs of society and the state of the discipline. The needs of society are very diverse: in each country, studies prepare for different professions, each of which has its own demands; between different countries there will be varying priorities. *A priori*, social needs introduce into curricula an element of diversity and even of divergence. On the other hand, reference to the discipline of mathematics itself is usually a unifying factor, when the specialists agree amongst themselves on what is essential content. And this unity also responds to a social need, to have a common body of knowledge and a shared language.

We have therefore to consider two major series of questions: the first relating to the expressed needs of society, to local experiences, to national policies; the second relating to new possibilities, to the adaptations which will have to be made as a result of new requirements, to choices prompted by the present state of knowledge and technique.

First we present three questions prompted by the social context (the national framework, the teaching of scientists, the industrial environment).

Question 1. In each country are there new mathematical curricula—permanent, provisional, experimental—motivated by the introduction of computers and informatics? The responses which we have so far received point to the existence of such experimental curricula.

Question 2. Mathematics has a duty to serve those in other disciplines—physicists, engineers, biologists, economists, etc. What are the changes prompted by the growing importance of computers and informatics within these disciplines? The partial replies we have received have come from the computer scientists themselves.

Question 3. What mathematics is necessary as a part of basic scientific culture—at a university level—within the new industrial environment? Those responses which we have had—coming from computer scientists—point to a strong theoretical demand; the use of computers and of informatics demand *more* mathematics, *better* understood, and would lead to a new equilibrium between "pure" and "applied" mathematics.

Let us pause at this point before going on to pose a new series of questions.

Doubtless, informatics will have three major effects on the orientation of teaching. First of all, symbolic mathematical systems are going to render simple and rapid, questions which were previously difficult and complicated. Already today there exist programs to evaluate definite integrals, to solve differential equations, even to calculate explicit solutions of certain functional equations. Thus mathematics teaching can lay less emphasis than formerly on the setting out and practising of classical methods of integration. On the other hand, our teaching can permit a student, by calling upon the available systems, to encounter a much greater number of problems and so understand better the underlying mathematics. The more such programs that will be at our disposal, the more necessary it will be for the student to understand the mathematical theory if (s)he is not to lose his/her bearings.

Next, informatics makes many calls upon the help of discrete mathematics: combinatorics, graph theory, coding theory. The applications of informatics to management, communication and information make little use of the differential and integral calculus, but they make use of varied structures on finite sets. It is advisable, then, to ask whether discrete mathematics should replace certain classical parts of analysis in the basic core of mathematics provided for students and whether certain fundamental concepts of analysis might not with advantage be approached *via* a study of discrete situations. For example, the place of series in analysis courses might need to be modified.

Finally, then, the general effect of computers and informatics on mathematics will have necessary consequences on its teaching, on the importance attached to subjects and to methods, and in the order chosen for the presentation of material.

In all the various branches of mathematics one can envisage computers supplying numerical and visual experiences intended to foster intuition. One can also favour algorithmic presentations of theories and proofs.

These thoughts lead us to pose a second set of questions:

Question 4. What is the mathematics underlying symbolic mathematical systems? How should they be introduced into the curriculum?

Question 5. What discrete mathematics should be introduced?

Question 6. What changes can be envisaged in the order of presentation of topics (series before integrals, statistics before probability, probability before integration, ...)?

Question 7. In particular, what elements of logic, numerical analysis, statistics, probability, geometry can be introduced from the very beginning of university courses?

Question 8. What are the foreseeable changes in the way individual topics are presented, particularly when one takes into account the available algorithms (Newton's method for solving equations, continued fractions for real numbers, polynomial interpolants in integration, triangulation in linear algebra, ...)?

Question 9 (Of major importance). What content might possibly be omitted in the foundation courses (17-18 years)?

The changes brought about on curricula by informatics and computers will obviously have consequences for the training needed by teachers. In addition to supplying the elements of computer science and informatics which they will need, we must also prepare them to teach mathematics in a new way. This problem is going to arise as much at the level of in-service training as at that of the initial (pre-service) training of teachers.

We therefore pose the following question:

Question 10. What elements of computer science and informatics should be introduced into the training of teachers, and how can they be prepared and helped to teach mathematics in a new way, consonant with the new computing context? Some experience in this area already exists.

3. The computer as an aid to the teaching of mathematics

3.1. The general effects of computers

The use of computers compels one not only to recognise in the area of experiments a source of mathematical ideas and a field for the illustration of results, but also a place where confrontation will permanently occur between theory and practice. This last poses a problem, which will occur in the training of teachers as well as of students, of promoting the *experimental attitude* (observation, testing, control of variables, ...) alongside, and on a par with, the *mathematical attitude* (conjecture, proof, verification, ...). Does it suffice, to speak, as some people do, of "experimental mathematics"?

We now have a triangle, student-teacher-computer, where previously only a dual relationship existed. Is there not a danger that, in order to preserve as much as possible the traditional student-teacher relationship, students' work on a computer will be restricted to simplistic activities which are "without risk" for the teacher?

Students are bound to be aware (as a result of their environment and the media) of the widespread use of computers as well as their associated peripherals, even interconnecting systems and data banks. They have also seen spectacular graphics displayed on a screen, or traced on a plotter. As a result of this, students have new expectations with respect to teaching in general and that of mathematics in particular. How can the computer be used by and with the students in order to meet these new expectations?

In addition to the changes of interest to which informatics leads, one must also draw attention to the changes in the difficulty of exercises and problems. Not only will the use of a computer change the order of difficulty of exercises, but it will also change the relative difficulties of the various ways of solving the same exercise. How can one arrive at new hierarchies and take them into account when one constructs exercises?

3.2. Objectives and modes of operation

There are various methods of using a computer in our teaching. The teacher can use the computer like a "blackboard", in the same way as one proceeds when giving demonstrations in the experimental sciences. However, the use of an interactive computer permits a much higher degree of interaction with the audience. This particular mode has been tested in various places, but its wider use depends upon the provision of more, and more varied, software. What are the specific requirements which must be met by such software?

The computer can be used by students, individually or in groups of two or more, in order to accomplish predetermined work (this is really programmed learning adapted to work on the computer: unfortunately, there seems to be little software of this type available having any great mathematical interest). In a similar manner, the computer can provide the student with a permanent and readily accessible form of self-evaluation.

Another use of the computer is for "practical work": the experimental manipulation of mathematical objects in connection with open-ended problems (e.g. statistical treatment of data, geometric explorations, the manipulation of functions, ...).

One sees, then, the need for the development of "software banks" in order both to provide support for teachers and lecturers and also to encourage further improvements. This software, which should be available to all, would be located in "multi-media centres" in the middle of institutions and seen as a means of communication on a par with written texts, films, ...

The preparation of software will call upon the united skills of mathematicians, computer scientists and practising teachers. How should one share out the work in order to produce satisfactory software and within which structural framework?

Finally, another use of the computer, in the school or university environment, is in the context of a "computer club". After an initial period of familiarisation, it is the users/members who are chiefly responsible for determining the paths to follow. This type of work is of relevance not only to students, but also to their teachers. What needs can be identified, therefore, for those who are going to be responsible for the training of teachers?

3.3. The treatment of particular areas

The peripherals used (screen, printer, plotter, ...) determine different ways of using informatics. The adaptation to mathematics poses some general problems, like that of the handling of symbolic writing which is not linear, despite the apparent linear sequence of characters in a normal text. For example, consider the various methods employed to reduce to a linear form mathematical statements often best presented in the form of a tree.

We now consider methods of employing the computer to meet needs to be found in various areas of mathematics which are taught at the levels of education under consideration.

In all of these branches one will note the central rôle of visualisation, of experimentation, of simulation, and of the way in which the computer fosters the generation and refining of conjectures.

First, however, we pose a general question. A certain number of fundamental concepts are used in the teaching of mathematics, often in an implicit manner, for example, intuitive logic, the concepts of a variable, of a function, ... Can informatics help us bring precision to, and increase our understanding of, such concepts?

Statistics and probability; data processing. The computer permits the processing of data on a truly grand scale. Problems of sorting data into classes are no longer relevant. Again, simulation is a tool which can hold a place in probability similar to that of plotting figures in geometry. Thus it is possible, thanks to pseudo-random techniques for selection, to provide "reality" for all types of conceivable situations—betting, decision making, testing ...

Geometry. The production of graphical images (e.g. perspective views of objects in space, orbits) and the concept of computer-aided design (graphics software) are extremely helpful for the development and fostering of intuitions. They make it possible to explore geometric objects and figures and provide access to new figures. What changes does plotting by means of a computer introduce with respect to geometry founded on the use of ruler and compass?

Linear algebra. The algorithmic approach furnishes tools for mathematical demonstrations (e.g. pivotal condensation) and leads us to approach in a different manner the study of such questions as inversion, the solution of systems of equations, and the decomposition of matrices. Moreover, visualisation can give support to intuition, e.g. for the study of eigenvalues and of diagonalisation. Do not such techniques as the simplex method merit a place in our teaching?

Analysis. As a result of the effects of symbolical systems, exercises on differentiation, searching for primitive integrals and finding finite Taylor series, are destined to decrease in importance. On the other hand, the graphical representation of functions and finding approximate solutions of numerical or functional equations will become worthy of additional consideration. Experimentation can provide opportunities for the discovery and formulation of qualitative properties before they are formally proved, for example, for the solution of differential equations. Approximation brings with it problems of convergence, beginning with sequences and series. Moreover, the qualitative aspect of the concept of convergence, the numerical study, leads naturally to the quantitative aspect, speed of convergence. Finally, discretisation provides a further field for experimentation, e.g. for functional equations.

Numbers, numerical analysis. The numbers of a machine are very different from those of a mathematician. This leads one to explore the differences and, *en passant*, to consider the principles of numerical symbolism. In another connection, should we be taking the use of parallel processors for research in numerical analysis into account at the teaching level?

Sets, combinatorics, logic. The methods of working now force one to give operative definitions (the enumeration of surjections $S(n, p)$ is a simple example of recursivity, which also allows one to give a working meaning to a surjection). In this area, too, the rapid production of numerical results permits easy exploration and the devising of conjectures. Does the learning of formulae by experience constitute a particular current interest in this field?

In traditional fields of study there are subjects which demand new and special attention because of the particular characteristics of working on a machine: that it uses discrete methods. It is important, therefore, to pay attention to theoretical approaches to discrete topics (e.g. difference equations); at this time, complete courses of discrete mathematics are being proposed for students. Is it really true that this provides us with a new theme for teaching?

3.4. Assessment and recording

The teacher often understands the assessment of his pupils' learning in the restricted sense of evaluation through examinations, while assessment of teaching is usually ignored. The computer, however, now makes possible a variety of ways of controlling assessment, ranging from the presentation of exercises to students to the management of individual files. The use of the computer to construct and to conduct evaluatory tests has hardly been experimented with up to now except in the teaching of computer science itself. Should we foresee a general development in the growth of examinations "on a computer", and if so how are such tests to be designed?

The notion of control and evaluation can also be extended to what happens when we use a computer. The juxtaposition of the output from a computer with mathematical results is specially relevant to such "experimental control". At the end of this report it is time to mention the usefulness of results which do not correspond to what has been foreseen and to those programs which do not function perfectly. It is obviously helpful to recall that frequently programs will not work at the first attempt. What is the mathematical interest in such mistakes?

3.5. The training of teachers

We have referred above to the problem of the content of teacher-training. It is equally advisable to question the form that this training should take, particularly the provision of in-service education for practising teachers. What can be envisaged if we think of "light" in-service training —day-release or short-term courses—and what if teachers can be given at least one year's complete leave of absence from teaching? But even this last is not sufficient considered in the context of the gradual evolution of materials and software. Here it would seem essential to open local support centres designed to provide a follow-up to such courses, to supply up-to-date software and to encourage teaching experiments. It would be a great pity if interest in computers and informatics resulted in the establishment of "heavy" administrative machinery, distant from most teachers, in which decisions relating to teaching were taken. What networks (local, regional, national, international) is it advisable, therefore, to set up and what type of connections must be established between them?