2. The effect of computers on curricula

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suitable for processors running in parallel. To take but one example, by mathematical ingenuity the time complexity for the fast Fourier transform algorithm has been reduced from n^2 to $n \log n$, which is of considerable practical importance for large values of n. There are other problems concerned with the effectiveness of algorithms, their correctness and the way in which they can be elaborated. We note, as an example, the rôle of invariants and fixed points when establishing the correctness of algorithms.

One must also stress how algorithms are increasingly being called upon to play a central rôle in society: they arise in business and commerce, in technology and in automation. Mathematical problems arise then in many new domains, and mathematical methods have an increasingly far-reaching applicability.

Finally, from now on symbolic systems will enable the computer user to carry out difficult calculations within algebra and analysis. The possibilities raised are enormous, and one must take the measure of the actual performance of such systems and of their rôle in research mathematics, as well as of the influence they should have on the teaching of mathematics at the university and pre-university levels. Informatics, for example, extends the field of mathematical research on formal calculus.

2. The effect of computers on curricula

Curricula are generally the product of a long tradition, and their evolution is governed by two principal factors: the needs of society and the state of the discipline. The needs of society are very diverse: in each country, studies prepare for different professions, each of which has its own demands; between different countries there will be varying priorities. *A priori*, social needs introduce into curricula an element of diversity and even of divergence. On the other hand, reference to the discipline of mathematics itself is usually a unifying factor, when the specialists agree amongst themselves on what is essential content. And this unity also responds to a social need, to have a common body of knowledge and a shared language.

We have therefore to consider two major series of questions: the first relating to the expressed needs of society, to local experiences, to national policies; the second relating to new possibilities, to the adaptations which will have to be made as a result of new requirements, to choices prompted by the present state of knowledge and technique.

First we present three questions prompted by the social context (the national framework, the teaching of scientists, the industrial environment).

Question 1. In each country are there new mathematical curricula—permanent, provisional, experimental—motivated by the introduction of computers and informatics? The responses which we have so far received point to the existence of such experimental curricula.

Question 2. Mathematics has a duty to serve those in other disciplines —physicists, engineers, biologists, economists, etc. What are the changes prompted by the growing importance of computers and informatics within these disciplines? The partial replies we have received have come from the computer scientists themselves.

Question 3. What mathematics is necessary as a part of basic scientific culture—at a university level—within the new industrial environment? Those responses which we have had—coming from computer scientists—point to a strong theoretical demand; the use of computers and of informatics demand *more* mathematics, *better* understood, and would lead to a new equilibrium between "pure" and "applied" mathematics.

Let us pause at this point before going on to pose a new series of questions.

Doubtless, informatics will have three major effects on the orientation of teaching. First of all, symbolic mathematical systems are going to render simple and rapid, questions which were previously difficult and complicated. Already today there exist programs to evaluate definite integrals, to solve differential equations, even to calculate explicit solutions of certain functional equations. Thus mathematics teaching can lay less emphasis than formerly on the setting out and practising of classical methods of integration. On the other hand, our teaching can permit a student, by calling upon the available systems, to encounter a much greater number of problems and so understand better the underlying mathematics. The more such programs that will be at our disposal, the more necessary it will be for the student to understand the mathematical theory if (s)he is not to lose his/her bearings.

Next, informatics makes many calls upon the help of discrete mathematics: combinatorics, graph theory, coding theory. The applications of informatics to management, communication and information make little use of the differential and integral calculus, but they make use of varied structures on finite sets. It is advisable, then, to ask whether discrete mathematics should replace certain classical parts of analysis in the basic core of mathematics provided for students and whether certain fundamental concepts of analysis might not with advantage be approached *via* a study of discrete situations. For example, the place of series in analysis courses might need to be modified.

Finally, then, the general effect of computers and informatics on mathematics will have necessary consequences on its teaching, on the importance attached to subjects and to methods, and in the order chosen for the presentation of material.

In all the various branches of mathematics one can envisage computers supplying numerical and visual experiences intended to foster intuition. One can also favour algorithmic presentations of theories and proofs.

These thoughts lead us to pose a second set of questions:

Question 4. What is the mathematics underlying symbolic mathematical systems? How should they be introduced into the curriculum?

Question 5. What discrete mathematics should be introduced?

Question 6. What changes can be envisaged in the order of presentation of topics (series before integrals, statistics before probability, probability before integration, ...)?

Question 7. In particular, what elements of logic, numerical analysis, statistics, probability, geometry can be introduced from the very beginning of university courses?

Question 8. What are the foreseeable changes in the way individual topics are presented, particularly when one takes into account the available algorithms (Newton's method for solving equations, continued fractions for real numbers, polynomial interpolants in integration, triangulation in linear algebra, ...)?

Question 9 (Of major importance). What content might possibly be omitted in the foundation courses (17-18 years)?

The changes brought about on curricula by informatics and computers will obviously have consequences for the training needed by teachers. In addition to supplying the elements of computer science and informatics which they will need, we must also prepare them to teach mathematics in a new way. This problem is going to arise as much at the level of in-service training as at that of the initial (pre-service) training of teachers.

We therefore pose the following question:

Question 10. What elements of computer science and informatics should be introduced into the training of teachers, and how can they be prepared and helped to teach mathematics in a new way, consonant with the new computing context? Some experience in this area already exists.

3. The computer as an aid to the teaching of mathematics

3.1. The general effects of computers

The use of computers compels one not only to recognise in the area of experiments a source of mathematical ideas and a field for the illustration of results, but also a place where confrontation will permanently occur between theory and practice. This last poses a problem, which will occur in the training of teachers as well as of students, of promoting the *experimental attitude* (observation, testing, control of variables, ...) alongside, and on a par with, the *mathematical attitude* (conjecture, proof, verification, ...). Does it suffice, to speak, as some people do, of "experimental mathematics"?

We now have a triangle, student-teacher-computer, where previously only a dual relationship existed. Is there not a danger that, in order to preserve as much as possible the traditional student-teacher relationship, students' work on a computer will be restricted to simplistic activities which are "without risk" for the teacher?

Students are bound to be aware (as a result of their environment and the media) of the widespread use of computers as well as their associated peripherals, even interconnecting systems and data banks. They have also seen spectacular graphics displayed on a screen, or traced on a plotter. As a result of this, students have new expectations with respect to teaching in general and that of mathematics in particular. How can the computer be used by and with the students in order to meet these new expectations?

In addition to the changes of interest to which informatics leads, one must also draw attention to the changes in the difficulty of exercises and problems. Not only will the use of a computer change the order of difficulty of exercises, but it will also change the relative difficulties of the various ways of solving the same exercise. How can one arrive at new hierarchies and take them into account when one constructs exercises?