5.5. *Remark.* From the preceding proof we can derive the upper bound $\exp\exp 4n^2$ for the number of isomorphism classes of n-dimensional crystallographic groups. The correct numbers for $n = 1, 2, 3, 4$ are respectively 2, 17, 219, 4783 [4].

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