# IX. Appendix: Orthomodular spaces over ordered fields

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THEOREM 34 [21]. *Let* $\mathfrak{E}$ *be a norm-topological space in the sense of Definition 31 and assume* $\varphi(2) = 0$. *Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.*

*Remark* 35. In Definition 15, Lemma 33 and in Theorem 34 we stipulated that $\varphi(2) = 0$ for the valuation $\varphi$ of the base field. However, it is neither necessary to assume this nor that char $k$ be different from two. As technicalities increase if 2 is not a unit for $\varphi$ the general case has been banned from this elementary survey. Refer to [21].

## IX. APPENDIX: ORTHOMODULAR SPACES OVER ORDERED FIELDS

A Baer order of a *-field $k$ is a subset $\Pi \subset S := \{\alpha \in k \mid \alpha = \alpha^*\}$ *with* $1 \in \Pi$, $0 \notin \Pi$, $\Pi + \Pi \subset \Pi$, $\forall\alpha \neq 0: \alpha\Pi\alpha^* \subset \Pi$, $-\Pi \,\dot\cup\, \Pi = S\backslash\{0\}$. ([14]). The map $\alpha \mapsto \alpha^*\alpha =: \| \alpha \|$ has the properties of a norm and defines a topology on $k$; if $*$ is continuous then $k$ is a topological *-field [14, Theorem 4.1, p. 231]. The theory of positive definite orthomodular spaces over archimedean ordered fields is settled in [9]: There are but the classical Hilbert spaces over **R**, **C**, **H**. If the order is non-archimedean we shall assume that

(15) the subgroup $S$ generated by all $\alpha^*\alpha^{-1}$ is bounded.

There is [14, Sec. 4.5, p. 234] a valuation on $k$ that induces the norm-topology. We remark that the boundness condition on $S$ is always satisfied for the usual orderings on commutative fields, for Prestel's semi-orderings and for all *-ordered fields that are known hitherto.

A family $(e_\iota)_{\iota\in I}$ of vectors in a *positive definite space* $(\mathfrak{E}; \langle\ ,\ \rangle)$ over an ordered *-field $k$ is said to satisfy the *type condition* (cf. Definition 21) iff for all $(\alpha_\iota)_{\iota\in I} \in k^I$ the following holds: if $(\langle\alpha_\iota e_\iota\rangle)_{\iota\in I}$ is bounded then $(\alpha_\iota e_\iota)_{\iota\in I}$ converges to $0 \in \mathfrak{E}$.

With this version of type condition we have

THEOREM 36. *Let* $(\mathfrak{E}; \langle\ ,\ \rangle)$ *be a positive definite space over a non-archimedean ordered *-field that satisfies (15). Then the statements (i), (ii), (iii) in Theorem 28 are equivalent.*