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So $\tilde{z} \in \tilde{N}$ as well since $\tilde{s}_1 \dots \hat{\tilde{s}}_i \dots \hat{\tilde{s}}_j \dots \tilde{s}_k \in \operatorname{Ker} \eta$, of length $\leqslant k - 2$ and *so* $\in \tilde{N}$. This gives a contradiction. Hence $\tilde{l}(\tilde{s}_i \dots \tilde{s}_j \cdot \tilde{s}_{j-1} \dots \tilde{s}_{i+1}) = k$ and $j = k = 2i$. Also, $s_1 \dots s_k = \mathrm{id} = s_1 \dots \hat{s}_i \dots \hat{s}_k$ and so $\tilde{s}_1 \dots \hat{\tilde{s}}_i \dots \hat{\tilde{s}}_k \in \tilde{N}$. Thus,

$$\tilde{z} \in \tilde{s}_1 \dots \tilde{s}_{i-1}\tilde{s}_i \dots \tilde{s}_1 \cdot \tilde{s}_k \cdot \tilde{N} .$$

Let $\tilde{z}_1 = \tilde{s}_k \cdot \tilde{s}_1 \dots \tilde{s}_{i-1} \cdot \tilde{s}_i \cdot \tilde{s}_{i-1} \dots \tilde{s}_1$ then $\tilde{z}_1 \in \tilde{z} \cdot \tilde{N}$ (Note that $\tilde{N}$ is normal).

Now argue with $\tilde{z}_1$ instead of $\tilde{z}$ (Note that $\tilde{l}(\tilde{z}_1) = k$ again!) Thus we get $\tilde{z}_2 = \tilde{s}_1\tilde{s}_k\tilde{s}_1 \dots \tilde{s}_{i-2}\tilde{s}_{i-1} \dots \tilde{s}_1 \cdot \tilde{s}_k \in \tilde{z}_1\tilde{N} = \tilde{z}\tilde{N}$ and so on. Finally, we get an element $\tilde{z}_r$ (for a suitable r) which is of the form $\tilde{s}_1\tilde{s}_k \dots \tilde{s}_1 \cdot \tilde{s}_k$ (total number of terms $= 2i$) and such that $\tilde{z}_r \in \tilde{z} \cdot \tilde{N}$. Since $\tilde{z}_r \in \operatorname{Ker} \eta$, it is clear that $m_{s_1, s_k} < \infty$ and it divides i and so $\tilde{z}_r \in \tilde{N}$ by definition. Thus $\tilde{z} \in \tilde{N}$ which is a contradiction. This finally proves that $\tilde{N} = \operatorname{Ker} \eta$ and so (1) holds.

This completes the proof of the main theorem.

REFERENCES

The references given here form a very small subset of a large literature available on Coxeter groups and related topics. Some of the references given are standard and some are included because of their need in the proof of main theorem.

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