since $|\chi| \leqslant 1$ and $|\xi/\theta| \leqslant \sqrt{3}$ for $(\xi/\theta) \in \operatorname{supp} \chi$; this gives the first estimate (4) with $C_{s,t} = 2^{s-t}$.

Similarly, for $s \leqslant t$,

$$(1+|\xi|^2)^s \,|\, \hat{v}(\xi) - \widehat{S_\theta v}(\xi) \,|^2 = |\, 1 - \chi(\xi/\theta) \,|^2 (1+|\xi|^2)^s \,|\, \hat{v}(\xi) \,|^2 ;$$

a Taylor formula gives $|\, 1 - \chi(\xi/\theta) \,| \leqslant C_k \,|\, \xi/\theta \,|^k$ with $C_k = \sup |\, \chi^{(k)} \,|/k!$ for any $k \in \mathbf{N}$ since $\chi(0) = 1$ and $\chi^{(j)}(0) = 0$ for $j > 0$, so that for $t = s + k$

$$\begin{aligned}(1+|\xi|^2)^s \,|\, \hat{v}(\xi) - \widehat{S_\theta v}(\xi) \,|^2 &\leqslant C_{t-s}^2 \,|\, \xi/\theta \,|^{2(t-s)} (1+|\xi|^2)^s \,|\, \hat{v}(\xi) \,|^2 \\ &\leqslant C_{t-s}^2 \theta^{2(s-t)} (1+|\xi|^2)^t \,|\, \hat{v}(\xi) \,|^2\end{aligned}$$

whence the second estimate (4) with $C_{s,t} = C_{t-s} = \sup |\, \chi^{(t-s)} \,|/(t-s)!$

## REFERENCES


[1] HAMILTON, R. The inverse function theorem of Nash-Moser. *Bulletin of the A.M.S. 7* (1982), 65-222.

[2] HÖRMANDER, L. The boundary problems of physical geodesy. *Arch. Rat. Mech. Anal. 62* (1976), 1-52.

[3] —— *Implicit function theorems.* Lectures at Stanford University, Summer Quarter 1977.

[4] —— On the Nash-Moser implicit function theorem. *Annales Acad. Sci. Fenniae, Series A.I. Math. 10* (1985), 255-259.

[5] MOSER, J. A new technique for the construction of solutions of nonlinear differential equations. *Proc. Nat. Acad. Sci. 47* (1961), 1824-1831.

[6] —— A rapidly convergent iteration method and nonlinear partial differential equations I and II. *Ann. Scuola Norm. Sup. di Pisa 20* (1966), 265-315 and 499-533.

[7] NASH, J. The imbedding problem for Riemannian manifolds. *Ann. of Math. 63* (1956), 20-63.

[8] SCHWARTZ, J. T. *Nonlinear functional analysis, Chap. II.A.* Gordon & Breach, New York 1969.

[9] SERGERAERT, F. Une généralisation du théorème des fonctions implicites de Nash. *C. R. Acad. Sci. Paris, 270A* (1970), 861-863.

[10] —— Un théorème des fonctions implicites sur certains espaces de Fréchet et quelques applications. *Ann. Sci. Ec. Norm. Sup. Paris 4e série, 5* (1972), 599-660.

[11] ZEHNDER, E. Generalized implicit function theorems with applications to some small divisor problems I and II. *Comm. in Pure and Appl. Math. 28* (1975), 91-140; *29* (1976), 49-111.


*(Reçu le 14 juin 1989)*

Xavier Saint Raymond

Purdue University and C.N.R.S.