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REPRESENTATION OF EVERY RATIONAL NUMBER AS AN ALGEBRAIC SUM OF FIFTH POWERS OF RATIONAL NUMBERS

by Ajai CHOUDHRY

The representation of a rational number as an algebraic sum of k^{th} powers of rational numbers has been considered by Subba Rao [1]. Let $g_1(k)$ be defined as the least integer s such that every rational number r can be expressed in the form

$$r = a_1 x_1^k + a_2 x_2^k + \dots + a_s x_s^k \tag{1}$$

where $a_i = \pm 1$ and all of the values of x_i are rational. It has been shown [1] that

$$g_1(5) \leqslant 8 .$$

In this note we shall prove that $g_1(5) \leqslant 6$. We shall also obtain a parametric solution of the Diophantine equation

$$x_1^5 + x_2^5 + \dots + x_s^5 = 0 \quad \text{for} \quad s \geqslant 6 . \tag{2}$$

The result follows from the identity

$$\begin{aligned} &(t^6x-t^{15}-t^5-1)^5 + (t^3x+t^2)^5 + (x-t^{14})^5 \\ &- (t^6x-t^{15}-t^5+1)^5 - (t^3x-2t^{12}-t^2)^5 - (x+t^{14})^5 \\ = &- 2(t^{40}+t^{30}-t^{10}-1)(20t^{11}x+t^{30}-12t^{20}-8t^{10}-1) . \end{aligned} \tag{3}$$

As the expression on the right-hand side of (3) is linear in x, it can be equated to any rational number r, and solved for x, which leads to the result

$$g_1(5) \leqslant 6 .$$

Thus, every rational number can be expressed as the algebraic sum of at most six fifth powers of rational numbers, and since in the above discussion, t can be taken as any rational number except that $t \neq 0, \pm 1$, this can be done in infinitely many ways.

To solve (2) for $s = 6$, we equate the right-hand side of (3) to 0, and obtain the result

$$\begin{aligned}&(t^{36}+8t^{26}+12t^{16}+20t^{11}-t^{6})^{5} + (t^{33}-12t^{23}-28t^{13}-t^{3})^{5}\\ &+ (t^{30}+20t^{25}-12t^{20}-8t^{10}-1)^{5} - (t^{36}+8t^{26}+12t^{16}-20t^{11}-t^{6})^{5}\\ &- (t^{33}+28t^{23}+12t^{13}-t^{3})^{5} - (t^{30}-20t^{25}-12t^{20}-8t^{10}-1)^{5} = 0\,.\end{aligned}$$

This result has earlier been given by Moessner [2].

To solve (2) for $s = 6 + m$, $m > 0$, we simply equate the right hand side of (3) to $\sum_{i=1}^{m} x_i^5$ where x_i are any rational numbers, and solve for x, which leads to a solution of (2) for $s > 6$. Solutions in integers can be obtained by multiplying by a suitable constant.

REFERENCES

[1] SUBBA RAO, K. Representation of Every Number as a Sum of Rational k^{th} Powers. *Journal London Math. Soc. 13* (1938), 14-16.

[2] MOESSNER, A. Due Sistemi Diofantei. *Boll. Un. Mat. Ital. Ser. 3, 6* (1951), 117-118.

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