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5. Linearization

5.1. The groups E_s^U can be viewed, through the homomorphism $\phi: E_s^U \to \pi_s(U)$ in 3.1, as "linear homotopy groups" of U. This means that we consider maps of S^s into U via some $U(n)$ which are linear in the coordinates $x_0, x_1, \ldots, x_s$ of $\mathbf{R}^{s+1} \supset S^s$; and linear nullhomotopies, i.e., extensions to $S^{s+1} \to U(n)$ linear in $x_0, x_1, \ldots, x_{s+1}$. It is an immediate corollary of Theorem B that these linear homotopy groups $\pi_s^{\text{lin}}(U)$ are isomorphic to the $\pi_s(U)$ by the obvious imbedding $\pi_s^{\text{lin}}(U) \to \pi_s(U)$. In other words:

Any map $S^s \to U$ is homotopic to a linear map, and if a linear map $S^s \to U$ is nullhomotopic then it admits a linear nullhomotopy.

Similar statements hold, of course, for $\pi_s(O)$ and $\pi_s(Sp)$.

5.2.If these linearization phenomena could be established directly (by some approximation procedure) one would obtain a very transparent proof of the Bott periodicity theorems for $\pi_s(U)$, $\pi_s(O)$, and $\pi_s(Sp)$, in the sense that they would be reduced to the algebraic computation of E_s^U, E_s^O, and E_s^{Sp} as carried out here.

5.3. Linear maps $S^s \to U$ via $U(n)$, etc., are given explicitly in terms of HR-matrices; thus the coefficients involve $0, \pm 1, \pm i$ only. Such maps have a meaning over very general fields instead of $\mathbf{R}$ and $\mathbf{C}$, and one should compare the corresponding linear homotopy groups with homotopy groups defined by means of algebraic maps.

REFERENCES

[E] Eckmann, B. Gruppentheoretischer Beweis des Satzes von Hurwitz-Radon über die Komposition quadratischer Formen. *Comment. Math. Helv. 15* (1942/43), 358-366.

[H] Hurwitz, A. Über die Komposition der quadratischen Formen. *Math. Ann. 88* (1923), 1-25.

[K] Karoubi, M. *K-Theory.* Springer-Verlag, Berlin-Heidelberg-New York 1978.

[LS] Lam, T. Y. and Tara Smith. On the Clifford-Littlewood-Eckmann groups: A new look at periodicity mod 8. Preprint Berkeley 1988.

[R] Radon, J. Lineare Scharen orthogonaler Matrizen. *Abh. Math. Sem. Hamburg* (1922), 1-14.

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