THE ROLE OF GAMES AND PUZZLES IN THE POPULARIZATION OF MATHEMATICS

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by Miguel de GUZMÁN

SUMMARY. The main features of popularization of mathematics are described first. Then a brief analysis of the relations between games and mathematics is presented. In the third place the uses of mathematical puzzles and games for the popularization of mathematics are indicated and finally attention is directed towards some objections to the use of mathematical games.

THE POPULARIZATION OF MATHEMATICS

In September 1989 the ICMI (International Commission on Mathematical Instruction) held an international meeting in Leeds (England) on the subject *The Popularization of Mathematics.* Almost one hundred participants from twenty different countries, including mathematicians, scientific journalists and writers of mathematics for a broad public gathered during a week in order to discuss the main features and problems of this activity called *the popularization of mathematics.* The main documents for study that arose for and from this meeting have been published by the ICMI [7] and by Cambridge University Press [4]. I shall borrow freely some of the very interesting ideas from these documents, above all from the introductory paper by Howson and Kahane, in order to better frame my own thinking about the role of games in this task of popularizing mathematics.

What is popularization?

Popularization is:

sharing mathematics, its beauty and its power, with a wider public, in some cases trying to demolish the prejudices and the traditional barriers between the *two cultures,* much sturdier around mathematics than around any other science;

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trying to change the attitudes toward mathematics of many who are in need of such a change, with the deep conviction that such attitudes are strongly harmful both for the development of mathematics and for the progress of human culture;

encouraging people to be more active mathematically, with the persuasion that this can lead them towards a more pleasurable and intellectually fuller life;

developing mathematics activity in freedom, not by compulsion, trying to get away from the odious feeling about mathematics that so many of the children and adults in our civilization share.

Why popularization?

1. Some people are driven to popularize:

by a sense of guilt, at the sight of the obvious failure of their work inside the educational system;

from a desire to survive from boredom, which is more deeply felt when one considers the richness of mathematics in contrast with the poverty of our educational programs;

out of a wish to make others participate in beauties of that part of mathematics that they can more clearly perceive, and this can happen at very many different levels;

because of the feedback one receives when one considers the strong effects of popularization on all kinds of people.

2. Some of the effects of popularization are:

more attention and support from different sources, when people in authority are able to better understand the connections of mathematics with many other aspects of our culture;

to bridge the existing gaps between the mathematical community and the ordinary citizen;

to increase the social status of mathematics within our civilization;

to improve the intellectual and cultural conditions of many people.

To whom should popularization be directed? What should be popularized? By whom?

Mathematics is a very complex structure. Very different kinds of persons and many types of mathematical activity could be profitably involved in its popularization, both in an active and a passive way:

Research mathematicians and university professors. They can be both transmitters and receivers of a certain kind of popularization, through expositions which could open a specific field to outsiders. Many new ideas appearing in one area can be very useful in neighbouring fields. Certain desirable conditions should be fulfilled: real reasonable accessibility to ideas without superfluous technicalities, from an expert who truly desires and knows how to communicate his knowledge to professional mathematicians unfamiliar with his field. In my opinion this is one of the most difficult tasks in the popularization of our mathematical knowledge. This difficulty and how far we are from the ideal are quite obvious when one attends our general International Congresses of Mathematicians and becomes aware of the failures of so many of our most outstanding mathematicians to give a moderately useful glimpse of what they have done to a wide audience of *professional* mathematicians. It is quite common to follow the easy path to talk to those (how many? perhaps 100 out of 4000?) who already know.

Teachers and university students. The general trends of contemporary mathematics should be made accessible to them by experts really competent both in the field and in the necessary skills to communicate at this level. Also the main highlights of the history and evolution of a field can be of great use to illuminate it for those who approach it for the first time.

Other professionals inside and outside the academic world. There are many aspects of mathematics they perhaps do not use in their work that could throw some new light on their way of thinking and solving their own problems.

High school students. The most important aspects of the history, evolution and applications of each one of the topics they are exposed to. The lifes of the most important men and women of mathematics. The cultural impacts of mathematics on the human history.

General public. Avoiding technicalities one should try to transmit as much as possible of the impact and methods of mathematical thinking about some special subjects. The biographies of dead and living mathematicians of deep interest. Applications, ideas and facts they should know as part of the human culture.

Small children. With a correct awareness of their possible interest and capacity, by those who know how to communicate with them with great enthusiasm. Through exhibits, competitions, games,...

GAMES AND MATHEMATICS

In the paper [3] I have tried to analyze in some detail the relationships between mathematics and games by looking at the intrinsic nature of these activities, at their respective modes of practice and at the ways they have impinged upon each other historically with great impact. Here I will briefly summarize some of the conclusions reached there in order to continue later with the implications this situation has for the popularization of mathematics.

Mathematics is a manysided activity:

Mathematics is a science, i.e. it is systematic knowledge; even more it is the paradigm and the oldest of all sciences.

Mathematics is an instrument for the exploration of the world, thanks to the systematic examination to which it subjects the different structures it detects in the universe and to its peculiar way of abstracting and expanding them.

Mathematics has been a model of thought, through its qualities of objectivity, consistency and soberness, which give it a preeminent place among the different human ways of acquiring information.

Mathematics is also an artistic activity, since it seeks to create a certain kind of beauty, intellectual beauty, only accessible, as Plato said, "to the eyes of the soul", and this is at the bottom perhaps the strongest driving and leading force present in the efforts of the greatest mathematicians.

But mathematics is also a game. When one looks at the main features by which sociologists characterize games (cf., for example, Johannes Huizinga's *Homo Ludens* [4]) one can find some traits that are also shared by many portions of mathematical activity as it has been performed throughout the centuries.

A game is characterized as

a free activity, free in the sense that it is cultivated by itself, not for the sake of other goals.

It performs a certain function in human development, and is certainly not solely and perhaps not mainly for children.

A game is not a joke, it has to be taken seriously, the worst game spoiler is the one who does not do this.

It generates pleasure through its execution and contemplation.

It is separated from ordinary life in time and in space.

It gives rise to special bonds among practitioners.

It creates a new order, a new life, full of rythm and harmony.

The structure of games and of mathematics, the way in which they *create a new order, a new life,* through the acceptance of certain objects and rules defining them and through the consistent adherence to this set of rules, is strikingly similar.

On the other hand, if one looks at the ways in which one is introduced, becomes acquainted and finally reaches a certain degree of expertise in games and in mathematics, one cannot fail to see a strong similarity, which is certainly not surprising when one considers the common features of games and mathematics in their nature and structure.

Given the strong similarities of mathematics and games, one should a priori expect a deep intertwining between them along history. The impact of games and their ludic spirit on mathematics and the presence of mathematics in games has in fact been very powerful.

It has very often been the case in the history of mathematics that an interesting question made in a gamelike manner or an ingenious observation about an apparently innocent situation has given rise to new modes of thinking. For an example of the strong impact of such a question in our century one can look at the expository paper [2] in *L'Enseignement Mathématique* on the mathematical developments around the innocent looking Kakeya problem (1919):

What is the plane figure of minimal area such that a needle of length one can be continuously inverted inside it?

Also one can see the strong and important implications of the game known as *The Tower of Hanoi* in an interesting paper by A. M. Hinz [5], which also appeared very recently in this journal. Similar examples can easily be found in ancient as well as in modern mathematics. The list of mathematical objects that have come to existence motivated by the spirit of games would certainly be without end.

If games have given rise to interesting, deep, useful and important mathematics, it is no less true that the richness of mathematical themes present in classical and modern games is impressive. The best way to perceive this is to look through the classical works by Lucas or Ball (Ball and Coxeter) and

through the bibliographical compilations made by W. L. Schaaf on the recent literature on games, published by the NCTM.

Still more interesting is the fact that there is so much deep mathematics with the flavour of games. Among modern examples one can select a few in which this is particularly obvious. Some of them can in fact be used as a basis for amusing and entertaining games: the four colour theorem, Ramsey's theorem, Sperner's lemma, the triangular billiards table, Helly's theorem, Hadwiger's conjecture,...

After these considerations it becomes quite easy to understand why so many among the greatest mathematicians have been so fond of games and in some cases have become great practitioners of some of them.

Fermat made very extensive and deep investigations on magic squares.

Leibniz writes in a letter to De Montmort (1715): "*Men are never more ingenious than in the invention of games; the spirit finds himself there at his leisure... it would be desirable to have a whole course about games mathematically treated*".

Euler writes in a letter to Goldbach (1744): "*Here one plays much chess. Among others there is a Jew who plays extraordinarily well. I have taken lessons from him and now I am so far that I win him most of the games*". And one can easily perceive that a good part of Euler's research has the flavor of mathematical games.

Many names of famous mathematicians could be mentioned here in connection with the ludic spirit of mathematics: Fibonacci, Cardano, Pascal, Daniel Bernoulli, Gauss, Hamilton, Hilbert, Einstein, von Neumann,...

GAMES AND PUZZLES AS A MEANS FOR POPULARIZATION

After what we have seen about the similarity in their intrinsic nature, structure, spirit and ways of practice of games and mathematics it should be quite clear that:

a game is very often good mathematics and frequently leads to deep and important mathematical developments;

playing a good game or puzzle is the closest thing to doing mathematics; it stimulates essentially the same abilities and often requires the same type of strategies as mathematics;

mathematics can be approached with the same ludic spirit of games and has been thus approached by many of the greatest mathematicians; this ludic and aesthetic spirit is at the very root of mathematical creativity.

But a game can avoid many of the pitfalls of solemn and serious mathematical presentations:

Games do not need boring systematic introductions before arriving to something interesting, as is often the case with our presentation of mathematics.

Good games and puzzles can avoid the effect of the psychological blocks that straight mathematical presentations tend to cause not only in children, but also in many adults, very often because of previous unpleasant mathematical experiences.

Games place everybody initially in a situation of equality, in which not so much depends on previous performance and knowledge as in mathematics.

Games are better at fostering ingenuity, imagination, phantasy, experimentation, manipulation,... since they constitute much more clearly a free activity quite open to us.

All these reasons can explain the great success of the really good writers of mathematical recreations along many centuries, and one can easily agree with Berlekamp, Conway and Guy, when in the dedication of their excellent work on games [1] they write:

> "To MARTIN GARDNER who has brought more mathematics to more millions than anyone else".

There are, of course, good and bad games and puzzles. What are the characteristics of a stimulating game from our point of view? I will point out some of the general traits that in my opinion help a game to be good:

attractive, beautiful, with a flavour of nearness and handiness...

easy to start on, no need of long and systematic introductions, can be approached from zero;

reasonably challenging, maybe looking easier than it really is;

perhaps with deep consequences for the real world, for the sciences, for mathematics,...

rooted in the particular culture, history of the people;

extendable, perhaps a whole world behind it...

its presentation should be made with genuine and contagious enthusiasm.

The great master of popularization of mathematics in the second half of our century, Martin Gardner, has assessed the situation quite rightly, talking about the use of games for mathematical teaching. The same could be said for popularization in general: "Surely the best way to wake up a student is to present him with an intriguing mathematical game, puzzle, magic trick, joke, paradox, limerick or any of a score of other things that dull teachers tend to avoid because they seem frivolous" (*Mathematical Carnival,* Preface).

Summarizing one could say that mathematics is a great and sophisticated game that, besides, happens to be an intellectual work of art bearing at the same time an intense light to explore the universe and so having great practical repercussions. The attempts to popularize mathematics through its applications, its history, the biography of the most interesting mathematicians, through the relationships with philosophy or other aspects of the human mind can serve very well to let mathematics be known by many persons. But possibly no other method can convey what is the right spirit of doing mathematics better than a well chosen game.

Finally I would like to present some of the more common objections that one encounters when dealing with games related to the popularization and the teaching of mathematics.

"I hate mathematical games".

There are many people, among them also mathematicians, who seem to be confused about the whole matter. There is a certain kind of puzzle and questions that are open to a great amount of ambiguity and trickery, in many cases intended to show you quite clearly how stupid you are. Such tricks can become noxious and harmful and they can cause a certain inferiority complex if one is overexposed to them. They give rise to a very false image of mathematics. Good mathematical games are not so. They are as well defined as any honest mathematical problem and their effect is to stimulate your ingenuity and imagination, not to let you down.

"I find them completely useless and inconsequential".

A little knowledge of the historical evolution of mathematics shows, as I have pointed out before, that many profound ideas of the greatest mathematicians could be traced down to their involvement in this kind of ludic thinking. It is rather difficult to draw the line between the important and the inconsequential. When presenting mathematical games to the great public one should take some

care to convey with them a certain respect towards the implications this kind of thinking can have and has really had along the history of science.

"It is a waste of time".

There is a difference between a healthy curiosity and dedication and a manic obsession that can absorb you for weeks without end. It is a matter of temperance and balance.

"It takes me far from my field".

Who knows? Perhaps an ingenious idea in a game can help you approach one of your problems from a new viewpoint. Sometimes it is very advisable to make excursions into neighbouring and even into distant fields, particularly when we get stuck in our work.

"Mathematical games lead our students to think that our mathematical life is something light and easy. When they find out, they shrink and go to something else".

There are games requiring different levels of skill and effort. One can easily find authentic research open problems among them. Many games can strongly attract people and stimulate them to do very hard work.

"Applied and useful mathematics is the real, serious enterprise. Pure mathematics is sterile. Approaching mathematics as a game is catastrophic".

I would like to end this paper with a quotation from Piet Hein *(Grooks)* that summarizes the whole of it and answers pretty well to this objection:

Taking fun
as simply fun
and earnestness
in earnest
shows how thoroughly
thou none
of the two discernest

REFERENCES

[1] BERLEKAMP, E. R., J. H. CONWAY and R. K. GUY. *Winning ways for your mathematical plays.* Academic Press, London, 1982.

[2] GUZMÁN, M. de. Some paradoxical sets with applications in the geometric theory of real variable. *L'Enseignement Math. (2) 29* (1983), 1-14.

[3] —— Games and Mathematics. In: A. G. Howson and J.-P. Kahane (editors), *The Popularization of Mathematics.* Cambridge Univ. Press, Cambridge, 1990.

[4] HOWSON, A. G. and J.-P. KAHANE (editors). *The Popularization of Mathematics.* Cambridge U. Press, Cambridge, 1990.

[5] HINZ, A. M. The Tower of Hanoi. *L'Enseignement Math. (2) 35* (1989), 289-321.

[6] HUIZINGA, J. *Homo ludens.* (1938) (Spanish transl.: Alianza Editorial, Madrid, 1972).

[7] ICMI Secretariat. *Papers on The Popularization of Mathematics.* Centre for Mathematical Education, Univ. of Southampton, 1989.

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Miguel de Guzmán

Facultad de Matemáticas
Universidad Complutense de Madrid
28040 Madrid