Games and mathematics

GAMES AND MATHEMATICS

In the paper [3] I have tried to analyze in some detail the relationships between mathematics and games by looking at the intrinsic nature of these activities, at their respective modes of practice and at the ways they have impinged upon each other historically with great impact. Here I will briefly summarize some of the conclusions reached there in order to continue later with the implications this situation has for the popularization of mathematics.

Mathematics is a manysided activity:

Mathematics is a science, i.e. it is systematic knowledge; even more it is the paradigm and the oldest of all sciences.

Mathematics is an instrument for the exploration of the world, thanks to the systematic examination to which it subjects the different structures it detects in the universe and to its peculiar way of abstracting and expanding them.

Mathematics has been a model of thought, through its qualities of objectivity, consistency and soberness, which give it a preeminent place among the different human ways of acquiring information.

Mathematics is also an artistic activity, since it seeks to create a certain kind of beauty, intellectual beauty, only accessible, as Plato said, "to the eyes of the soul", and this is at the bottom perhaps the strongest driving and leading force present in the efforts of the greatest mathematicians.

But mathematics is also a game. When one looks at the main features by which sociologists characterize games (cf., for example, Johannes Huizinga's *Homo Ludens* [4]) one can find some traits that are also shared by many portions of mathematical activity as it has been performed throughout the centuries.

A game is characterized as

a free activity, free in the sense that it is cultivated by itself, not for the sake of other goals.

It performs a certain function in human development, and is certainly not solely and perhaps not mainly for children.

A game is not a joke, it has to be taken seriously, the worst game spoiler is the one who does not do this.

It generates pleasure through its execution and contemplation.

It is separated from ordinary life in time and in space.

It gives rise to special bonds among practitioners.

It creates a new order, a new life, full of rythm and harmony.

The structure of games and of mathematics, the way in which they *create a new order, a new life,* through the acceptance of certain objects and rules defining them and through the consistent adherence to this set of rules, is strikingly similar.

On the other hand, if one looks at the ways in which one is introduced, becomes acquainted and finally reaches a certain degree of expertise in games and in mathematics, one cannot fail to see a strong similarity, which is certainly not surprising when one considers the common features of games and mathematics in their nature and structure.

Given the strong similarities of mathematics and games, one should a priori expect a deep intertwining between them along history. The impact of games and their ludic spirit on mathematics and the presence of mathematics in games has in fact been very powerful.

It has very often been the case in the history of mathematics that an interesting question made in a gamelike manner or an ingenious observation about an apparently innocent situation has given rise to new modes of thinking. For an example of the strong impact of such a question in our century one can look at the expository paper [2] in *L'Enseignement Mathématique* on the mathematical developments around the innocent looking Kakeya problem (1919):

What is the plane figure of minimal area such that a needle of length one can be continuously inverted inside it?

Also one can see the strong and important implications of the game known as *The Tower of Hanoi* in an interesting paper by A. M. Hinz [5], which also appeared very recently in this journal. Similar examples can easily be found in ancient as well as in modern mathematics. The list of mathematical objects that have come to existence motivated by the spirit of games would certainly be without end.

If games have given rise to interesting, deep, useful and important mathematics, it is no less true that the richness of mathematical themes present in classical and modern games is impressive. The best way to perceive this is to look through the classical works by Lucas or Ball (Ball and Coxeter) and

through the bibliographical compilations made by W. L. Schaaf on the recent literature on games, published by the NCTM.

Still more interesting is the fact that there is so much deep mathematics with the flavour of games. Among modern examples one can select a few in which this is particularly obvious. Some of them can in fact be used as a basis for amusing and entertaining games: the four colour theorem, Ramsey's theorem, Sperner's lemma, the triangular billiards table, Helly's theorem, Hadwiger's conjecture,...

After these considerations it becomes quite easy to understand why so many among the greatest mathematicians have been so fond of games and in some cases have become great practitioners of some of them.

Fermat made very extensive and deep investigations on magic squares.

Leibniz writes in a letter to De Montmort (1715): "*Men are never more ingenious than in the invention of games; the spirit finds himself there at his leisure... it would be desirable to have a whole course about games mathematically treated*".

Euler writes in a letter to Goldbach (1744): "*Here one plays much chess. Among others there is a Jew who plays extraordinarily well. I have taken lessons from him and now I am so far that I win him most of the games*". And one can easily perceive that a good part of Euler's research has the flavor of mathematical games.

Many names of famous mathematicians could be mentioned here in connection with the ludic spirit of mathematics: Fibonacci, Cardano, Pascal, Daniel Bernoulli, Gauss, Hamilton, Hilbert, Einstein, von Neumann,...

GAMES AND PUZZLES AS A MEANS FOR POPULARIZATION

After what we have seen about the similarity in their intrinsic nature, structure, spirit and ways of practice of games and mathematics it should be quite clear that:

a game is very often good mathematics and frequently leads to deep and important mathematical developments;

playing a good game or puzzle is the closest thing to doing mathematics; it stimulates essentially the same abilities and often requires the same type of strategies as mathematics;