According to the theorem of Schinzel and Tijdeman, for every $m \geqslant 2$ there exists $C_m > 0$ (and effectively computable) such that if $(x, y, z)$ is a non-trivial solution of $(E_m)$, then $|x|, y, z < C_m$.

LEMMA 7. *With above hypotheses, if* $m \geqslant 2$ *and* $k > C_m$, *then* $(E_{km})$ *has only trivial solutions.*

*Proof.* Let $k > C_m$, let $(x, y, z)$ be a non-trivial solution of $(E_{km})$. So $f(x^{km}) = y^z$, thus $(x^k, y, z)$ is a non-trivial solution of $(E_m)$. Hence $|x^k| < C_m < k$. Then $|x| \leqslant 1$ and therefore $f(0), f(1)$ or $f(-1)$ is a proper power, which is contrary to the hypothesis. $\square$

Let $F = \{m \geqslant 2 \mid (E_m)$ has only trivial solutions$\}$.

PROPOSITION 8. $\mu(F) = 1$.

*Proof.* Let $F'$ be the complement of the set $F$; it suffices to show that $\mu(F') = 0$.

For each prime $p$, $kp \in F$ for each $k > C_p$, according to lemma 7. So $\mathbf{N}p \cap F'$ is finite. By lemma 2, $\mu(F') = 0$. $\square$

Actually, the complement of $F$ is finite, if $f$ has at least two simple zeroes.

We note the following interesting application. Let $a, b, c$ be non-zero integers, such that $-\dfrac{c}{b}$ and $\pm\dfrac{a}{b} - \dfrac{c}{b}$ are not zero, nor 1, nor proper powers.

Let $f = \dfrac{a}{b}X - \dfrac{c}{b}$. The above result is applicable to the polynomial $f$ and yields:

*The set of* $m \geqslant 3$ *such that there exist integers* $n \geqslant 2$, *and* $x, y$, *with* $y \geqslant 2$, *satisfying* $ax^m - by^n = c$, *has uniform density equal to zero.*

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*(Reçu le 28 janvier 1992)*

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