

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 39 (1993)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: QUICK LOWER BOUNDS FOR REGULATORS OF NUMBER FIELDS
Autor: Skoruppa, Nils-Peter

Kurzfassung

DOI: <https://doi.org/10.5169/seals-60417>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 22.12.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

QUICK LOWER BOUNDS FOR REGULATORS OF NUMBER FIELDS

by Nils-Peter SKORUPPA

ABSTRACT. A short and simple proof for Zimmert's lower bounds for regulators of number fields is presented.

1. INTRODUCTION. Let K be an algebraic number field with r_1 real and $2r_2$ complex embeddings, let R denote its regulator and w its number of roots of unity. The purpose of this note is to present a surprisingly short proof of the following theorem.

THEOREM. For any real number $s > 1$ one has

$$\frac{R}{w} \geq \frac{s(s-1)}{e} \exp\left(-\frac{s}{s-1}\right) \gamma(s) \exp\left(-s \frac{\gamma'(s)}{\gamma}\right),$$

Here $\gamma(s) = 2^{-r_1} \Gamma(s/2)^{r_1} \Gamma(s)^{r_2}$, where $\Gamma(s)$ denotes the gamma function, and $\gamma'(s)$ is the derivative.

If we let $s = 4/3$ then we obtain

$$\frac{R}{w} \geq 0.00299 \cdot \exp(0.48r_1 + 0.06r_2).$$

From this one deduces that regulators of number fields are bounded from below by an absolute constant and grow exponentially in the degree of K .

This result and an estimate similar to the one of the above theorem was first stated and proved by Zimmert [Z, Satz 3 and Korollar]. He proved the sharper estimate

$$\frac{R}{w} \geq \frac{s(2s-1)}{2e} \exp\left(-\frac{2s}{s-1}\right) \gamma(2s) \exp\left(-2s \frac{\gamma'(s)}{\gamma}\right)$$