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Finally we like to show that the non-emptiness condition on the index cone of a projective 3-fold with $h^{0,2} = 0$ gives non-trivial restrictions for the possible cup-forms if $b_2 \geq 4$. Further investigations of this condition will appear elsewhere [Sch].

EXAMPLE 17. Let H be a free \mathbf{Z} -module of rank 4 with basis $(e_i)_{i=1,\dots,4}$. Consider a trilinear form $F \in S^3 H^\vee$ and its adjoint map $F^t: H \rightarrow S^2 H^\vee$. The image $F^t(h)$ of an element $h \in H$ is in terms of the chosen basis $(e_i)_{i=1,\dots,4}$ represented by the symmetric 4×4 -matrix $[[he_i e_j]]_{i,j=1,\dots,4}$. Suppose this matrix is a diagonal sum $[[he_i e_j]]_{i,j=1,2} \oplus [[he_k e_l]]_{k,l=3,4}$ such that the determinants of both 2×2 -matrices are negative for every $h \in H \setminus \{0\}$.

In this case $F^t(h)$ were of signature $(1, -1, 1, -1)$ for every $h \in H \setminus \{0\}$, and we would have $I_F = \mathcal{H}_F = \emptyset$.

All these conditions can be met, e.g. by setting $e_1^2 e_2 = e_2^3 = e_3^2 e_4 = e_4^3 = 1$, $e_1 e_2^2 = e_3 e_4^2 = 2$, and $e_i e_j e_k = 0$ otherwise. In this particular case the image of $h = \sum_{i=1}^4 h_i e_i$ under F^t is represented by the matrix

$$\left[\begin{array}{cc|cc} h_2 & h_1 + 2h_2 & & \\ h_1 + 2h_2 & 2h_1 + h_2 & & \\ \hline & & h_4 & h_3 + 2h_4 \\ & 0 & h_3 + 2h_4 & 2h_3 + h_4 \end{array} \right],$$

which has a positive determinant unless $h = 0$.

REFERENCES

- [A] ARNDT, F. Zur Theorie der binären kubischen Formen. *Crelle's Journal* 53, 309-321 (1857).
- [Ar1] ARONHOLD, S. Zur Theorie der homogenen Funktionen dritten Grades von drei Variablen. *Crelle's Journal* 39, 140-159 (1850).
- [Ar2] ——— Theorie der homogenen Funktionen dritten Grades von drei Veränderlichen. *Crelle's Journal* 55, 93-191 (1858).
- [At] ATIYAH, M. Some examples of complex manifolds. *Bonner Math. Schriften* 5, 1-28 (1958).
- [A/H/S] ATIYAH, M., N. HITCHIN and I. SINGER. Self-duality in four-dimensional Riemannian geometry. *Proc. R. Soc. Lond. A.* 362, 425-461 (1978).

- [B] BEAUVILLE, A. Variétés Kählériennes dont la première classe de Chern est nulle. *J. Diff. Geom.* 18, 755-782 (1983).
- [Bo] BOREL, A. *Introduction aux groupes arithmétiques*. Publ. de L'Institut Math. de L'Univ. de Strasbourg. Hermann, Paris 1969.
- [B/H-C] BOREL, A., HARISH-CHANDRA. Arithmetic subgroups of algebraic groups. *Annals of Math. (2)* 75, 485-535 (1962).
- [B/H] BOREL, A. and F. HIRZEBRUCH. Characteristic classes and homogeneous spaces, III. *Amer. J. Math.* 82, 491-504 (1960).
- [B/V] BRIESKORN, E. and A. VAN DE VEN. Some complex structures on products of homotopy spheres. *Topology* 7, 389-393 (1968).
- [C/E] CALABI, E. and B. ECKMANN. A class of compact, complex manifolds which are not algebraic. *Ann. of Math.* 58, 494-500 (1959).
- [C] CAMPANA, F. On Twistor Spaces of the class C. *J. Diff. Geom.* 33, 541-549 (1991).
- [C/P] CAMPANA, F. and T. PETERNELL. Rigidity theorems for primitive Fano 3-folds. Preprint, Bayreuth 1991.
- [Ca] CASSELS, J. *An Introduction to the Geometry of Numbers*. Grundlehren Bd. 99, Springer, Berlin 1959.
- [Cay] CAYLEY, A. Tables of the binary cubic forms for the negative discriminants, $\equiv 0 \pmod{4}$, from -4 to -400 ; and $\equiv 1 \pmod{4}$, from -3 to -99 ; and for five irregular negative determinants. *The Quart. J.* 11, 246-261 (1871).
- [Co] CORNALBA, M. Una osservazione sulla topologia dei rivestimenti ciclici di varietà algebriche. *Bolletino U.M.I. (5)* 18-A, 323-328 (1981).
- [D/G/M] DELIGNE, P., P. GRIFFITHS and J. MORGAN. Real homotopy theory of Kähler manifolds. *Invent. math.* 29, 245-274 (1975).
- [D] DICKSON, L. *History of the theory of numbers. Vol. III. Quadratic and higher forms*. Reprinted by: Chelsea Publishing Company; New York, N.Y. 1971.
- [D/F] DONALDSON, S. and R. FRIEDMANN. Connected sums of self-dual manifolds and deformations of singular spaces. *Nonlinearity* 2, 197-239 (1989).
- [E/L1] EIN, L. and R. LAZARSELD. Syzygies and Koszul cohomology of smooth projective varieties of arbitrary dimension. *Invent. math.* 111, 373-392 (1993).
- [E/L2] EIN, L. and R. LAZARSELD. Global generation of pluricanonical and adjoint linear series on smooth projective threefolds. Preprint, 1992.
- [F] FRIEDMANN, R. On threefolds with trivial canonical bundle. *Proc. Symp. Pure Math.* 53, 103-134 (1991).
- [F/M] FRIEDMANN, R. and J. MORGAN. *Smooth four-manifolds and complex surfaces*. Ergeb. Math. 3. Folge, Springer, Heidelberg 1994.
- [G/N] GORDAN, P. and M. NOETHER. Über die algebraischen Formen, deren Hesse'sche Determinante identisch verschwindet. *Math. Ann.* 10, 547-568 (1876).
- [G/H] GRIFFITHS, P. and J. HARRIS. *Principles of Algebraic Geometry*. J. Wiley and Sons, New York 1978.
- [G/S] GRUNEWALD, F. and D. SEGAL. Some general algorithms. I: Arithmetic groups. *Ann. of Math.* 112, 531-583 (1980).
- [H] HILBERT, D. Über die vollen Invariantensysteme. *Math. Ann.* 42, 313-373 (1893).

- [H1] HIRZEBRUCH, F. Komplexe Mannigfaltigkeiten. In: *Proc. Int. Congress of Math. 1958*, 119-136. Cambridge University Press, 1960.
- [H2] ——— Some examples of threefolds with trivial canonical bundle. Notes by J. Werner. Preprint, MPI Bonn 1985.
- [Hi] HITCHIN, N. Kählerian Twistor Spaces. *Proc. London Math. Soc.* 43, 133-150 (1981).
- [J1] JORDAN, C. Sur l'équivalence des formes algébriques. *C. R. Acad. Sc. XC*, 1422-1423 (1880).
- [J2] ——— Mémoire sur l'équivalence des formes. *J. Ec. Polytechn. XLVIII*, 111-150 (1880).
- [J] JUPP, P. Classification of certain 6-manifolds. *Proc. Camb. Phil. Soc.* 73, 293-300 (1973).
- [K1] KATO, M. Examples of simply connected compact complex 3-folds. *Tokyo J. Math.* 5, 341-364 (1982).
- [K2] ——— On compact complex 3-folds with lines. *Japan J. Math.* 11, 1-58 (1985).
- [K/Y] KATO, M. and A. YAMADA. Examples of simply connected complex 3-folds II. *Tokyo J. Math.* 9, 1-28 (1986).
- [K/M/M] KAWAMATA, Y., K. MATSUDA and K. MATSUKI. Introduction to the minimal model problem. *Adv. Stud. Pure Math.* 10, 283-360 (1987).
- [Ko1] KOLLÁR, J. Flips, flops, minimal models, etc. *Survey in Diff. Geom.* 1, 113-199 (1991).
- [Ko2] KOLLÁR, J. Effective base point freeness. Preprint, Utah 1992.
- [Kr] KRAFT, H. *Geometrische Methoden in der Invariantentheorie*. Aspekte der Math., Vieweg, Braunschweig 1984.
- [L] LAMOTKE, K. The topology of complex projective varieties after S. Lefschetz. *Topology* 20, 15-51 (1981).
- [L/S] LANTERI, A. and D. STRUPPA. Projective manifolds with the same homology as \mathbf{P}_k . *Monatsh. Math.* 101, 53-58 (1986).
- [L/W] LIBGOBER, A. and J. WOOD. Differentiable structures on complete intersections - I. *Topology* 21, 469-482 (1982).
- [M] MAEDA, H. Some complex structures on the product of spheres. *J. Fac. Sci. Univ. Tokyo* 21, 161-165 (1974).
- [Mi] MIYAOKA, Y. The Chern classes and Kodaira dimension of a minimal variety. *Adv. Stud. Pure Math.* 10, 449-476 (1987).
- [Mo] MORI, S. Threefolds whose canonical bundle is not numerically effective. *Ann. Math.* 116, 133-176 (1982).
- [M/M1] MORI, S. and S. MUKAI. On Fano 3-folds with $b_2 \geq 2$. *Manus. math.* 36, 147-162 (1981).
- [M/M2] MORI, S. and S. MUKAI. Classification of Fano 3-folds with $b_2 \geq 2$. *Adv. Studies Pure Math.* 1, 101-129 (1981).
- [M/F] MUMFORD, D. and J. FOGARTY. *Geometric invariant theory*. 2nd ed. *Ergeb. Math.* 34, Springer, Heidelberg 1982.
- [Mu] MURRE, J. *Classification of Fano threefolds according to Fano and Iskovskikh*. Springer Lecture Notes in Mathematics 947, 35-92.
- [N] NEWSTEAD, P. *Introduction to moduli problems and orbit spaces*. Tata Inst. Lecture Notes, Springer, Heidelberg 1978.
- [O1] OGUIISO, K. On polarized Calabi-Yau 3-folds. *J. Fac. Sci. Univ. Tokyo* 38, 395-429 (1991).

- [O2] — On algebraic fiber space structures on a Calabi-Yau 3-fold. Preprint, Bonn 1992.
- [O3] — A note on the graded ring of a polarized Calabi-Yau 3-fold. Preprint, Bonn 1993.
- [O4] — Two remarks on Moishezon Calabi-Yau 3-folds. Preprint, Bonn 1993.
- [O/V] OKONEK, Ch. and A. VAN DE VEN. *Stable Bundles, Instantons and C^∞ -structures on Algebraic Surfaces*. Encyclopaedia of Math. Sciences, Vol. 69, 198-249, Springer, Berlin-Heidelberg 1990.
- [P] POINCARÉ, H. Formes cubiques ternaires et quaternaires. *J. Ec. Polytechn.* 51, 45- 91 (1882).
- [S] SESHADRI, C. Geometric Reductivity over Arbitrary Base. *Advances in Math.* 26, 225-274 (1977).
- [Sch] SCHMITT, A. Zur Topologie dreidimensionaler komplexer Mannigfaltigkeiten. Ph. D. thesis, Zürich 1995.
- [Si] SIEGEL, C. The integer solutions of the equation $y^2 = ax^n + bx^{n-1} + k$. *J. London Math. Soc.* 1, 66-68 (1926).
- [Sm] SMALE, S. On the structure of manifolds. *Amer. J. Math.* 84, 387-399 (1962).
- [St] STURMFELS, B. *Algorithms in Invariant Theory*. Springer, New York 1993.
- [W] WALL, C.T.C. Classification problems in differential topology, V. On certain 6-Manifolds. *Invent. math.* 1, 355-374 (1966).
- [We] WERNER, J. Neue Beispiele dreidimensionaler Varietäten mit $c_1 = 0$. *Math. Gottingensis* 20, 1988.
- [Wi] WILSON, P. Calabi-Yau manifolds with large Picard number. *Invent. math.* 98, 139-155 (1989).
- [Y] YAU, S. On the Ricci curvature of a compact Kähler manifold and the complex Monge-Ampère équation, I. *Comm. Pure & Appl. Math.* 31, 339-411 (1978).
- [Z1] ŽUBR, A. Classification of simply-connected six-dimensional Spin-manifolds. *Izv. Akad. Nauk SSSR, Ser. Mat.*, 39, 793-812 (1975).
- [Z2] — Classification of simply connected six-dimensional manifolds. *Dokl. Akad. Nauk SSSR*, 255, 828-831 (1980).
- [Z3] — Classification of simply-connected topological 6-manifolds. (*Rohlin-volume*) *LNM 1346*, 325-339 (1988).

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