0. Introduction

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CHARACTERISTIC CLASSES, ELLIPTIC OPERATORS AND COMPACT GROUP ACTIONS

by James L. HEITSCH [1])

0. INTRODUCTION

Our aim in this paper is to introduce characteristic classes of vector bundles, to relate them to the indices of elliptic operators and the Lefschetz theorems for such operators, and finally to show how to use these to prove the non-existence of non-trivial actions preserving a Spin structure for compact connected Lie groups. Specifically, we will show how to prove the following.

THEOREM 5.2 ([HL 2]). *Let F be an oriented foliation of a compact oriented manifold M and assume that F admits a Spin structure. If a compact connected Lie group acts non-trivially on M as a group of isometries taking each leaf of F to itself and preserving the Spin structure on F, then the $\widehat{A}$ genus of F is zero.*

In [HL 1] and [HL 2], we assumed that F admitted a transverse invariant measure. In this paper, we show how to remove that rather restrictive hypothesis by employing the Haefliger forms of F. In particular, the traces we use here have values in those forms rather than in the complex numbers. A transverse invariant measure defines a map from the Haefliger zero forms to the reals, and applying it to the traces we use in this paper produces the traces used in [HL 1] and [HL 2]. Note in particular that all the results of [HL 1] and [HL 2] are still valid even if F does not admit a transverse invariant measure. One need merely ignore the transverse invariant measure and interpret the traces used as taking values in the Haefliger zero forms instead of the complex

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numbers. For another application of Haefliger forms and their cohomology, see [He].

An immediate corollary is the following theorem of Atiyah and Hirzebruch.

THEOREM 5.3 ([AH]). *Let M be a compact connected, oriented manifold which admits a Spin structure. If a compact connected Lie group acts non-trivially on M, then the $\widehat{A}$ genus of M is zero.*

Theorem 5.2 is an application of the Lefschetz fixed point theorem for complexes elliptic along the leaves of a foliated manifold. We explain the classical Lefschetz theorem for elliptic complexes and give an outline of how to prove it. The original proofs of this theorem relied on the fact that the underlying manifold was compact. We outline a proof which does not rely on that fact, and so can be generalized to complexes defined along the leaves of a compact foliated manifold. Note that such leaves are in general not compact, but the fact that they come from a foliation of a compact manifold means that they have uniformly bounded geometry. It is this property which allows us to prove the foliation version of the Lefschetz theorem. We then show how the Lefschetz theorem leads to Theorem 5.2. Finally, we give a brief explanation of a very general rigidity theorem conjectured by Witten and proven by Bott and Taubes.

This paper is based on lectures given at the conference Actions Différentiables de Groupes Compacts, Espaces d'Orbites et Classes Caractéristiques, held at the Université des Sciences et Techniques du Languedoc in Montpellier in January, 1994. The author wishes to thank the organizers, especially Daniel Lehmann and Pierre Molino, for extending the invitation to him to speak at the conference and for making his stay in Montpellier so pleasant.

1. CHARACTERISTIC CLASSES AND MULTIPLICATIVE SEQUENCES

All objects considered in this paper will be smooth. Let E be an n dimensional complex vector bundle over the real manifold M. Denote the space of smooth sections of E by $C^\infty(E)$. A connection on E is a linear map $\nabla : C^\infty(E) \to C^\infty(T^*M \otimes E)$ satisfying

$$\nabla(f \cdot \sigma) = df \otimes \sigma + f \cdot \nabla\sigma$$

for any $\sigma \in C^\infty(E)$ and $f \in C^\infty(M)$, the smooth functions on M. T^*M denotes the cotangent bundle of M.