that $\mathcal{L} \leq \frac{2\pi}{\sqrt{K}}$, otherwise $\mathcal{L}$ is too large for γ to be a circle and the desired inequality holds trivially. Under these conditions inequality (1) is equivalent to

$$\mathcal{A} \leq \frac{1}{K}\left\{2\pi - \sqrt{4\pi^2 - K\mathcal{L}^2}\right\}. \tag{17}$$

Combining Theorem B and (17) we obtain

$$\mathcal{L} \geq \frac{1}{\sqrt{K}} \tan\left(\frac{\sqrt{K}\mathcal{W}}{2}\right)\sqrt{4\pi^2 - K\mathcal{L}^2},$$

which is equivalent to

$$\mathcal{L} \geq \frac{2\pi}{\sqrt{K}} \sin\left(\frac{\sqrt{K}\mathcal{W}}{2}\right) \tag{18}$$

— and this is the inequality we want. If equality holds in (18) then it also holds in each of the equivalent inequalities (17) and (1) — and therefore γ is a circle.

The case $K < 0$ has a similar (and easier) treatment. We begin by rewriting (1) in the form

$$\mathcal{A} \leq -\frac{1}{K}\left\{\sqrt{4\pi^2 - K\mathcal{L}^2} - 2\pi\right\},$$

and then proceed as before. □

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