Thus the characteristic and the difference $\omega - 2\pi v$ remain constant when one "cuts off a corner". And finally, since $\mathcal{P}'$ is again convex, we can now iterate and end up with a tetrahedron, for which both theorems are clear. So the proofs go through in the convex case, and Euler was on the right track after all.

A last comment on convexity: In the figures in Euler's two papers all the vertex cones appear to be convex. (The vertex cone of a vertex V of a polyhedron $\mathcal{P}$ consists of all rays from V through points of $\mathcal{P}$ "near" V.) This property would make the whole polyhedron convex (see Bonnesen-Fenchel [1], p. 3, "Konvexität im Kleinen"). Of course this wasn't known in Euler's time (nor was the question even raised); but he might have felt that it was obvious.

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