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ON THE GAUSS-BONNET FORMULA FOR LOCALLY SYMMETRIC SPACES OF NONCOMPACT TYPE

by Enrico LEUZINGER

ABSTRACT. Let X be a Riemannian symmetric space of noncompact type and rank ≥ 2 and let Γ be a non-uniform, irreducible lattice in the group of isometries of X. A Gauss-Bonnet formula for the locally symmetric quotient $V = \Gamma \backslash X$ was first proved by G. Harder. We present a new simple proof which is based on an exhaustion of V by Riemannian polyhedra with uniformly bounded second fundamental forms.

INTRODUCTION

The generalized Gauss-Bonnet theorem of C. B. Allendoerfer, A. Weil and S. S. Chern asserts that the Euler characteristic of a *closed* Riemannian manifold (M, g) is given by

$$\chi(M) = \int_M \omega_g$$

where the Gauss-Bonnet-Chern form $\omega_g = \Psi_g \, dv_g$ is (locally) computable from the metric g (see [AW], [C]).

In several articles J. Cheeger and M. Gromov investigated the Gauss-Bonnet theorem for *open* complete Riemannian manifolds with bounded sectional curvature and finite volume. They in particular showed that such manifolds M^n admit an exhaustion by compact manifolds with smooth boundary, M_i^n, such that $\mathrm{Vol}(\partial M_i^n) \to 0$ $(i \to \infty)$ and for which the second fundamental forms $\mathrm{II}(\partial M_i^n)$ are uniformly bounded (see [CG1], [CG2], [CG3] and also [G] 4.5.C). By a formula of Chern one has $\chi(M_i^n) = \int_{M_i^n} \omega_g + \int_{\partial M_i^n} \eta_i$ where η_i is a certain form on the boundary ∂M_i^n (see [C]). The above two properties imply that $\lim_{i \to \infty} \int_{\partial M_i^n} \eta_i = 0$ and hence $\chi(M_i^n) = \int_{M^n} \omega_g$ for sufficiently large i. As a consequence the Gauss-Bonnet theorem holds whenever $\chi(M_i^n) = \chi(M^n)$ for all sufficiently large i.

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