1. The formula of Allendoerfer and Weil

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **42 (1996)**

Heft 3-4: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **09.08.2024**

1. The formula of Allendoerfer and Weil

A C^∞ (resp. C^ω) manifold with corners is a topological Hausdorff space locally modeled upon a product of lines and half-lines and such that coordinate changes are of class C^∞ (resp. C^ω). For precise definitions and basic information about this concept we refer to [DH]. A *Riemannian polyhedron* is a compact manifold with corners equipped with a Riemannian metric.

Let $\mathcal{P}^n$ be an n-dimensional Riemannian polyhedron with boundary consisting of a finite family of lower dimensional subpolyhedra

$$\mathcal{P}_E^{n-k} \ (0 \le k \le n-1)$$

and with Riemannian metric induced from $\mathcal{P}^n$. The *outer angle* $O(p)$ at a point p of $\mathcal{P}_E^{n-k}$ is defined as the set of all unit tangent vectors $v \in T_p\mathcal{P}^n$ such that $\langle v, w\rangle_p \le 0$ for all w in the tangent cone of $\mathcal{P}^n$ at p. Note that $O(p)$ is a spherical cell bounded by "great spheres" in the $(k-1)$-dimensional unit sphere of the normal space of $\mathcal{P}_E^{n-k} \subset \mathcal{P}^n$ at p. In [AW] Allendoerfer and Weil define a certain real valued function $\Psi_{E,k}$ on the outer angles of $\mathcal{P}_E^{n-k}$. The explicit form of this function will not be needed in this paper. We shall only use the fact that $\Psi_{E,k}$ is locally computable from the components of the metric and the curvature tensor of $\mathcal{P}^n$ and from the components of the second fundamental forms $\mathrm{II}_Z(p), Z \in O(p)$, of $\mathcal{P}_E^{n-k}$ in $\mathcal{P}^n$. Let Ψdv denote the Gauss-Bonnet-Chern form on $\mathcal{P}^n$ and dv_E (resp. $d\omega_{k-1}$) the volume element of $\mathcal{P}_E^k$ (resp. of the standard unit sphere S^{k-1}). The *inner Euler characteristic* χ' of $\mathcal{P}^n$ is by definition the Euler characteristic of the open complex consisting of all inner cells in an arbitrary simplicial subdivision of $\mathcal{P}^n$.

We can now state the generalized Gauss-Bonnet formula of Allendoerfer-Weil for Riemannian polyhedra (see [AW]).

Proposition 1.1. *Let $\mathcal{P}^n$ be a Riemannian polyhedron with boundary consisting of a finite family of subpolyhedra $\mathcal{P}_E^{n-k}$. Then the inner Euler characteristic of $\mathcal{P}^n$ is given by*

$$(-1)^n\chi'(\mathcal{P}^n) = \int_{\mathcal{P}^n} \Psi dv + \sum_{k=1}^{n}\sum_{E}\int_{\mathcal{P}_E^{n-k}} \Big(\int_{O(p)} \Psi_{E,k}\, d\omega_{k-1}\Big)\, dv_E(p)\,.$$