Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 43 (1997)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ATKIN-LEHNER EIGENFORMS AND STRONGLY MODULAR LATTICES
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Kapitel: Introduction
DOI: https://doi.org/10.5169/seals-63271

L'Enseignement Mathématique, t. 43 (1997), p. 55–65

ATKIN-LEHNER EIGENFORMS AND STRONGLY MODULAR LATTICES

by H.-G. QUEBBEMANN

SUMMARY. There are two arithmetical objects associated with any triple (k, ℓ, χ) consisting of an even positive integer k, a squarefree positive integer ℓ, and a character χ of the group of Atkin-Lehner involutions on $\Gamma_0(\ell)$ which maps the Fricke involution to $(-1)^{k/2}$. Namely, there is a space of common Atkin-Lehner eigenforms of weight k, and there is a genus of positive definite lattices in dimension $2k$. In general, Siegel's weighted mean of the theta series from the latter genus lies in the former space. For an individual lattice, however, the same holds under the condition of strong modularity introduced in this paper. Many interesting lattices known in higher-dimensional euclidean space are strongly modular, and their theta series are explained by the theory of modular forms.

INTRODUCTION

This paper is mainly concerned with lattices on euclidean n-space that are even and similar to their duals. Let the similarity norm be ℓ and $n = 2k$. Then the theta function of such a lattice is a modular form of weight k on $\Gamma_0(\ell)$ and an eigenform of the Fricke operator. (This was used in [Qu] for primes ℓ.) When the level ℓ is composite, however, one must also take care of invariance with respect to the other Atkin-Lehner involutions. This will be our subject here.

The algebraic structure of the relevant Atkin-Lehner eigenforms turns out to be most simple when the sum of the positive divisors of ℓ divides 24. As a consequence, the notion of extremal lattices introduced by Mallows, Odlyzko and Sloane for $\ell = 1$ ([CS], Ch.7) applies in a uniform way to $\ell = 1, 2, 3, 5, 6, 7, 11, 14, 15, 23$. This explains, in particular, the theta series of quite a few remarkable lattices occurring in recent work by Nebe and Plesken ([N1], [NP]). In fact, their material has been a main stimulus to the present study.