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AN ASYMPTOTIC FREIHEITSSATZ  
FOR FINITELY GENERATED GROUPS

by Pierre-Alain CHERIX\*) and Gilles SCHAEFFER

ABSTRACT. Given two fixed integers  $k \geq 2$  and  $l \geq 3$ , let  $\Gamma = \langle X \mid R \rangle$  be a presentation of the group  $\Gamma$  with  $k = \#X$  generators and  $l = \#R$  relations. We show that the following property of presentations of groups is generic in the sense of Gromov: for any  $y \in X$ , the subgroup of  $\Gamma$  generated by  $X - \{y\}$  is free of rank  $k - 1$ . This gives some generic estimates for the spectral radius of the adjacency operator in the Cayley graph of  $\Gamma$  relative to the generating system  $S = X \cup X^{-1}$ .

1. INTRODUCTION

The existence of free subgroups in some finitely generated group  $\Gamma$  gives some information about the structure of  $\Gamma$ . For example, it implies that  $\Gamma$  is non-amenable, and in particular that  $\Gamma$  has exponential growth. There are several results which ensure that various groups do have non-abelian free subgroups. For example:

THEOREM (Tits's alternative [15]). *Let  $\Gamma$  be a finitely generated linear group. Then either  $\Gamma$  is almost solvable or  $\Gamma$  contains a free subgroup on two generators.*

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THEOREM (Magnus's Freiheitssatz [12]). *Let  $\Gamma = \langle X|r \rangle$  be a one relator group,  $x_0 \in X$  be a generator of  $\Gamma$  that appears in the relation  $r$  and  $r$  be a cyclically reduced word in the free group  $\mathbf{F}_X$  generated by  $X$ ; then  $X - \{x_0\}$  freely generates a free group in  $\Gamma$ .*

Our purpose in this work is to measure in some sense how frequent it is for a presentation  $\Gamma = \langle X|R \rangle$  to be such that a proper subset of  $X$  is free in  $\Gamma$ . We prove the following result:

THEOREM 1.1. *Let  $\Gamma = \langle X|R \rangle$  be a finite presentation with  $k$  generators,  $l$  relations and any fixed  $x_0$  in  $X$ . Then the fact that  $X - \{x_0\}$  freely generates a free group in  $\Gamma$  is generic in the sense of Gromov.*

The key idea is contained in proposition 4.1. Roughly speaking, if you choose at random  $l$  long relations and if the presentation satisfies a Dehn algorithm, then every generator  $x_0$  will appear in every sufficiently long subword of every relation and hence it will appear in every product of conjugates of relations. So  $X - \{x_0\}$  generates a free group in  $\Gamma$ .

In [6], the first author has shown that “ $X$  generates a free semi-group” is generic and that this implies bounds on the spectrum of the adjacency operator associated to the oriented Cayley graph of  $\Gamma$  relative to  $X$ . In section 5 below, we consider the adjacency operator  $h_S$  of the Cayley graph of  $\Gamma$  relative to  $S = X \cup X^{-1}$ , and we prove similarly estimates on the norm of  $h_S$ .

After finishing this paper we discovered that a result similar to Theorem 1.1 has been proved, using different methods, by G. Arzhantseva and A. Ol'shanskii in [1]. They employed a slightly different definition of the genericity and they proved that the small cancellation condition  $C'(\lambda)$  is generic with respect to this new definition.

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