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THE REPRESENTATION THEORY OF AFFINE TEMPERLEY-LIEB ALGEBRAS

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ABSTRACT. We define a sequence $\mathbf{T}^a(n)$ $(n = 0, 1, 2, 3, \ldots)$ of infinite dimensional algebras as the sets of endomorphisms of the objects in a certain category of diagrams. These algebras are extended versions of the Temperley-Lieb quotients of the affine Hecke algebras of type $\widetilde{A}_{n-1}$. They have bases consisting of diagrams drawn without intersections on the surface of a cylinder. Using the methods of cellular algebras, we construct certain finite dimensional representations of these algebras, which we call "cell" or "Weyl" modules; these come from "functors on the category of diagrams" and are therefore constructed simultaneously for all $\mathbf{T}^a(n)$.

There are canonical invariant bilinear forms which put pairs of the cell modules in duality with each other and all the irreducible $\mathbf{T}^a(n)$-modules are obtained as quotients of the cell modules by the radicals of the forms. By determining all homomorphisms between the cell modules, we are able to determine their decomposition matrices and from these to deduce the dimensions of all the irreducibles.

We also give explicit formulae for the discriminants of the forms. The representations we construct may be interpreted as representations of the affine Hecke algebra of type A; they therefore give explicit results about some of the representations of the affine Hecke algebra at roots of unity. Our results may also be applied to study related finite dimensional algebras such as the usual Temperley-Lieb algebra or Jones' annular algebra. For these, our results concerning discriminants give precise criteria for semisimplicity as well as a complete discussion of their modular representation theory, including the determination of the composition factors, with their multiplicities, of the cell modules. As a by-product of our explicit determination of the homomorphisms between the cell modules, we also obtain a closed formula for the Jones (or augmentation) idempotent of the Temperley-Lieb algebra which yields a presentation of Jones' projection algebra when the Jones trace on the Temperley-Lieb algebra is degenerate.