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Artikel: RECENT DEVELOPMENTS ON SERRE'S MULTIPLICITY CONJECTURES: GABBER'S PROOF OF THE NONNEGATIVITY CONJECTURE

Bibliographie

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We remark that the construction we have presented is quite computational in the sense that it is possible to compute the embedding ϕ explicitly in special cases. We give two simple examples. First, let R have dimension 2 with maximal ideal generated by t, u, let $A = R[X, Y]$, and let I be generated by $uX - tY$. Then $A_0 = k[X, Y]$. The map f to $A_0[S_1, S_2, T_0, T_1]$ induced by the partial derivatives sends $uX - tY$ to $-YS_1 + XS_2 + uT_0 - tT_1$, which, after dividing by $\mathfrak{m}$, is $-YS_1 + XS_2$. Let $\mathfrak{p} = (t, u)$. Then $uX - tY$ is zero modulo $\mathfrak{p}$, so the kernel on the map of graded rings is generated by the image of $uX - tY$ in I/I^2. Hence $\mathcal{M}$ is mapped to the sheaf associated to $A_0[S_1, S_2, T_0, T_1]/(-YS_1 + XS_2)$. It can be verified that this quotient satisfies the condition on Hilbert polynomials; the positivity condition also follows from the fact that $-YS_1 + XS_2$ has degree $(1, 1)$.

Finally, we consider the example from section 3 in which I is generated by $t^2 - u^3, uX - tY, X^2 - uY^2$. Then I/I^2 has rank 2. Taking derivatives, we see that the map ϕ (after dividing by $\mathfrak{m}$) satisfies $\phi(t^2 - u^3) = 0$, $\phi(uX - tY) = XS_1 - TS_2$, and $\phi(X^2 - uY^2) = -Y^2S_2 + 2XT_0$. To compute the result of intersecting with Y', where Y' is generated by an ideal $\mathfrak{p}$, it suffices to compute the kernel of the map from the symmetric algebra on I/I^2 to the associated graded ring of I on $R/\mathfrak{p}[X, Y]$ tensored with k, and then find the image of this kernel in $A_0[S_1, S_2, T_0, T_1]$. On the other hand, in this case $\mathrm{Proj}(A_0) = \mathrm{Proj}(k[X, Y]/(X^2))$ has dimension zero, so that the locally free sheaf defined by $(I/I^2) \otimes_R k$ is actually positive.

Similar examples can be computed from the other examples in section 3.

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