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## HOMFLY POLYNOMIAL VIA AN INVARIANT OF COLORED PLANE GRAPHS

by Hitoshi MURAKAMI, Tomotada OHTSUKI and Shuji YAMADA

ABSTRACT. After the first discovery of the quantum invariant associated with  $SU(2)$  by V.F.R. Jones [3], the invariants associated with  $SU(n)$  were found by several authors [1]. It was first proved by V.G. Turaev [16] that all these come from so-called “quantum groups”, especially from their  $R$ -matrices corresponding to the vector representations. There also exist various quantum invariants corresponding to other representations (see for example [7], [14], [11]).

The aim of this paper is to give a graphical way to define  $SU(n)$  quantum invariants for links. To do this we first construct an invariant of colored, oriented, trivalent, plane graphs for each  $n$  ( $\geq 2$ ). Then we show that the  $SU(n)$  polynomial invariant corresponding to the vector representation (HOMFLY polynomial) can be defined by using our graph invariant.

We can also show that our invariant defines the  $SU(n)$  polynomial invariant corresponding to the anti-symmetric tensors of the vector representation.

We note that our graph invariant for  $SU(3)$  was first introduced by G. Kuperberg in [8]. The second and the third authors used it in [12] to construct magic elements and defined the quantum  $SU(3)$  invariants for 3-manifolds. Now [12] and the present paper together give an elementary and self-contained proof of the existence of magic elements for  $SU(3)$  and so that of the quantum  $SU(3)$  invariants of 3-manifolds just like W.B.R. Lickorish did for  $SU(2)$  in [9] using the Kauffman bracket [5]. See [17] for a similar approach to  $SU(n)$  invariants of 3-manifolds.

We also note that our graph invariant may be obtained (not checked yet) by direct computations of the universal  $R$ -matrix. But the advantage of our definition is that it does not require any knowledge of quantum groups nor representation theory. On the contrary we can recover the  $R$ -matrix of the quantum group  $U_q(\mathfrak{sl}(n, \mathbb{C}))$  corresponding at least to the vector representation.

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