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B. TABLE OF THE INVARIANT

The following is the table of $P_n/[n]$ for knots with crossings less than or equal to eight. (Note that P_n is always divisible by $[n]$.) We refer the reader to [15] for the notations of knots. We used MAPLE V to translate the Homfly polynomial table in [10] to our invariants.

3_1	$-q^{2n} + q^{n+1} + q^{n-1}$
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4_1	$q^n - q + 1 - q^{-1} + q^{-n}$
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5_1	$-q^{3n+1} - q^{3n-1} + q^{2n+2} + q^{2n} + q^{2n-2}$
5_2	$-q^{3n} + q^{2n+1} - q^{2n} + q^{2n-1} + q^{n+1} - q^n + q^{n-1}$

6_1	$q^{2n} - q^{n+1} + q^n - q^{n-1} - q + 2 - q^{-1} + q^{-n}$
6_2	$q^{2n+1} - q^{2n} + q^{2n-1} - q^{n+2} + q^{n+1} - 2q^n + q^{n-1} - q^{n-2} + q + q^{-1}$
6_3	$-q^{n+1} + q^n - q^{n-1} + q^2 - q + 3 - q^{-1} + q^{-2} - q^{-n+1} + q^{-n} - q^{-n-1}$

7_1	$-q^{4n+2} - q^{4n} - q^{4n-2} + q^{3n+3} + q^{3n+1} + q^{3n-1} + q^{3n-3}$
7_2	$-q^{4n} + q^{3n+1} - q^{3n} + q^{3n-1} + q^{2n+1} - 2q^{2n} + q^{2n-1} + q^{n+1} - q^n + q^{n-1}$
7_3	$q^{-2n+2} - q^{-2n+1} + q^{-2n} - q^{-2n-1} + q^{-2n-2} + q^{-3n+2} - q^{-3n+1} + 2q^{-3n}$ $- q^{-3n-1} + q^{-3n-2} - q^{-4n+1} - q^{-4n-1}$
7_4	$q^{-n+1} - 2q^{-n} + q^{-n-1} + 2q^{-2n+1} - 2q^{-2n} + 2q^{-2n-1} + q^{-3n+1} - 2q^{-3n}$ $+ q^{-3n-1} - q^{-4n}$
7_5	$-q^{4n+1} + q^{4n} - q^{4n-1} + q^{3n+2} - 2q^{3n+1} + 2q^{3n} - 2q^{3n-1} + q^{3n-2} + q^{2n+2}$ $- q^{2n+1} + 2q^{2n} - q^{2n-1} + q^{2n-2}$
7_6	$-q^{3n} + 2q^{2n+1} - 2q^{2n} + 2q^{2n-1} - q^{n+2} + 2q^{n+1} - 3q^n + 2q^{n-1} - q^{n-2}$ $+ q - 1 + q^{-1}$
7_7	$-q^{n+1} + 2q^n - q^{n-1} + q^2 - 2q + 4 - 2q^{-1} + q^{-2} - 2q^{-n+1} + 2q^{-n}$ $- 2q^{-n-1} + q^{-2n}$

8 ₁	$q^{3n} - q^{2n+1} + q^{2n} - q^{2n-1} - q^{n+1} + 2q^n - q^{n-1} - q + 2 - q^{-1} + q^{-n}$
8 ₂	$q^{3n+2} - q^{3n+1} + q^{3n} - q^{3n-1} + q^{3n-2} - q^{2n+3} + q^{2n+2} - 2q^{2n+1} + q^{2n}$ $- 2q^{2n-1} + q^{2n-2} - q^{2n-3} + q^{n+2} + q^n + q^{n-2}$
8 ₃	$q^{2n} - 2q + 3 - 2q^{-1} - q^{n+1} + 2q^n - q^{n-1} - q^{-n+1} + 2q^{-n} - q^{-n-1} + q^{-2n}$
8 ₄	$q^{2n+1} - q^{2n} + q^{2n-1} - q^{n+2} + 2q^{n+1} - 2q^n + 2q^{n-1} - q^{n-2} - q^2 + q - 2$ $+ q^{-1} - q^{-2} + q^{-n+1} + q^{-n-1}$
8 ₅	$q^{-n+2} + 2q^{-n} + q^{-n-2} - q^{-2n+3} + q^{-2n+2} - 3q^{-2n+1} + q^{-2n} - 3q^{-2n-1}$ $+ q^{-2n-2} - q^{-2n-3} + q^{-3n+2} - q^{-3n+1} + 2q^{-3n} - q^{-3n-1} + q^{-3n-2}$
8 ₆	$q^{3n+1} - q^{3n} + q^{3n-1} - q^{2n+2} + 2q^{2n+1} - 3q^{2n} + 2q^{2n-1} - q^{2n-2} - q^{n+2}$ $+ 2q^{n+1} - 3q^n + 2q^{n-1} - q^{n-2} + q + q^{-1}$
8 ₇	$-q^2 + q - 1 + q^{-1} - q^{-2} + q^{-n+3} - q^{-n+2} + 3q^{-n+1} - 2q^{-n} + 3q^{-n-1}$ $- q^{-n-2} + q^{-n-3} - q^{-2n+2} + q^{-2n+1} - 2q^{-2n} + q^{-2n-1} - q^{-2n-2}$
8 ₈	$-q^{n+1} + q^n - q^{n-1} + q^2 - 2q + 4 - 2q^{-1} + q^{-2} + q^{-n+2} - 2q^{-n+1}$ $+ 3q^{-n} - 2q^{-n-1} + q^{-n-2} - q^{-2n+1} + q^{-2n} - q^{-2n-1}$
8 ₉	$q^{n+2} - q^{n+1} + 2q^n - q^{n-1} + q^{n-2} - q^3 + q^2 - 3q + 3 - 3q^{-1} + q^{-2} - q^{-3}$ $+ q^{-n+2} - q^{-n+1} + 2q^{-n} - q^{-n-1} + q^{-n-2}$
8 ₁₀	$-q^2 + q - 2 + q^{-1} - q^{-2} + q^{-n+3} - q^{-n+2} + 4q^{-n+1} - 2q^{-n} + 4q^{-n-1}$ $- q^{-n-2} + q^{-n-3} - q^{-2n+2} + q^{-2n+1} - 3q^{-2n} + q^{-2n-1} - q^{-2n-2}$
8 ₁₁	$q^{3n+1} - q^{3n} + q^{3n-1} - q^{2n+2} + 2q^{2n+1} - 4q^{2n} + 2q^{2n-1} - q^{2n-2} + q - 1$ $+ q^{-1} - q^{n+2} + 3q^{n+1} - 3q^n + 3q^{n-1} - q^{n-2}$
8 ₁₂	$-2q^{n+1} + q^{2n} - 2q^{n-1} + 3q^n + q^2 - 3q + 5 - 3q^{-1} + q^{-2} - 2q^{-n+1}$ $+ 3q^{-n} - 2q^{-n-1} + q^{-2n}$
8 ₁₃	$-q^{2n+1} + q^{2n} - q^{2n-1} + q^{n+2} - 2q^{n+1} + 4q^n - 2q^{n-1} + q^{n-2} - q^{-n+1}$ $+ 2q^{-n} - q^{-n-1} + q^2 - 3q + 4 - 3q^{-1} + q^{-2}$
8 ₁₄	$q^{3n+1} - 2q^{3n} + q^{3n-1} - q^{2n+2} + 3q^{2n+1} - 4q^{2n} + 3q^{2n-1} - q^{2n-2} - q^{n+2}$ $+ 3q^{n+1} - 4q^n + 3q^{n-1} - q^{n-2} + q - 1 + q^{-1}$
8 ₁₅	$q^{5n} - 3q^{4n+1} + 2q^{4n} - 3q^{4n-1} + 2q^{3n+2} - 3q^{3n+1} + 5q^{3n} - 3q^{3n-1} + 2q^{3n-2}$ $+ q^{2n+2} - 2q^{2n+1} + 3q^{2n} - 2q^{2n-1} + q^{2n-2}$
8 ₁₆	$-q^{2n+2} + 2q^{2n+1} - 3q^{2n} + 2q^{2n-1} - q^{2n-2} + q^{n+3} - 2q^{n+2} + 4q^{n+1} - 4q^n$ $+ 4q^{n-1} - 2q^{n-2} + q^{n-3} - q^2 + 2q - 2 + 2q^{-1} - q^{-2}$

817	$q^{n+2} - 2q^{n+1} + 3q^n - 2q^{n-1} + q^{n-2} - q^3 + 2q^2 - 4q + 5 - 4q^{-1} + 2q^{-2} - q^{-3} + q^{-n+2} - 2q^{-n+1} + 3q^{-n} - 2q^{-n-1} + q^{-n-2}$
818	$q^{n+2} - 3q^{n+1} + 3q^n - 3q^{n-1} + q^{n-2} - q^3 + 3q^2 - 4q + 7 - 4q^{-1} + 3q^{-2} - q^{-3} + q^{-n+2} - 3q^{-n+1} + 3q^{-n} - 3q^{-n-1} + q^{-n-2}$
819	$q^{-3n+3} + q^{-3n+1} + q^{-3n} + q^{-3n-1} + q^{-3n-3} - q^{-4n+2} - q^{-4n+1} - q^{-4n} - q^{-4n-1} - q^{-4n-2} + q^{-5n}$
820	$-q^{2n+1} - q^{2n-1} + q^{n+2} + 2q^n + q^{n-2} - q + 1 - q^{-1}$
821	$q^{3n+1} - q^{3n} + q^{3n-1} - q^{2n+2} + q^{2n+1} - 3q^{2n} + q^{2n-1} - q^{2n-2} + 2q^{n+1} - q^n + 2q^{n-1}$

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