Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 44 (1998)

Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE ORIGIN OF REPRESENTATION THEORY

Autor: CONRAD, Keith

Bibliographie

DOI: https://doi.org/10.5169/seals-63909

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 23.12.2024

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

irreducible representations of a finite group, a theorem of Brauer says this number equals the number of conjugacy classes in the group consisting of elements with order prime to p.

7. Recent results

Character tables do not provide a way to distinguish any two finite groups, in general. For example, for any prime p the two nonisomorphic nonabelian groups of order p^3 have the same character table. Can we find a computational tool extending the character table which will distinguish any two non-isomorphic finite groups? In 1991, Formanek and Sibley [19] showed that if there is a bijection between two groups G and H which converts $\Theta(G)$ to $\Theta(H)$, then G and H are isomorphic. Since the irreducible characters can be read off (in principle) from the factors of $\Theta(G)$, we see $\Theta(G)$ is one answer to the question. However, if $\#G$ is large then $\Theta(G)$ is too hard to compute. Is there something closer to the character table which works? Yes. See the articles of Hoehnke and Johnson [28, 29] and Johnson and Sehgal [31].

Acknowledgements. I thank T. Geisser, K. Khuri-Makdisi, T. Klenke, T. Y. Lam, A. Ofer, R. Pollack, J.-P. Serre, and R. Vakil for their assistance, and MSRI for its hospitality, where part of this was written.

References

[1] Artin, E. Über eine neue Art von L-Reihen. *Math. Ann. 89* (1923), 89–108.

[2] Borevich, Z. I. and I. R. Shafarevich. *Number Theory.* Academic Press, New York, 1966.

[3] Burnside, W. On a property of certain determinants. *Mess. of Math. (N.S.) 23* (1894), 112–114.

[4] —— On the condition of reducibility of any group of linear substitutions. *Proc. London Math. Soc. (2) 3* (1905), 430–434.

[5] Catalan, E. Recherches sur les déterminants. *Bulletins de l'Académie Royale des sciences, des lettres et des beaux-arts de Belgique 13* (1846), 534–555.

[6] Cremona, L. Intorno ad un teorema di Abel. *Annali di Scienze matematiche e fisiche 7* (1856), 99–105.

[7] Curtis, C. Representation theory of finite groups: from Frobenius to Brauer. *Math. Intelligencer 14* (1992), 48–57.

[8] Curtis, C. and I. Reiner. *Methods of Representation Theory, Vol. 1.* John Wiley & Sons, New York, 1981.

[9] DAVENPORT, H. Bases for finite fields. *J. London Math. Soc. 43* (1968), 21–39.

[10] DEDEKIND, R. *Gesammelte mathematische Werke, Vol. II.* Chelsea, New York, 1969.

[11] DICKSON, L.E. An elementary exposition of Frobenius' theory of group characters and group-determinants. *Ann. of Math. (2) 4* (1902), 25–49; also *Mathematical Papers, Vol. II.* Chelsea, New York, 1975, 737–761.

[12] —— The order of a certain senary linear group. *Amer. Math. Monthly 9* (1902), 149–152; also *Mathematical Papers, Vol. VI.* Chelsea, New York, 1983, 477–480.

[13] —— A matrix defined by the quaternion group. *Amer. Math. Monthly 9* (1902), 243–248; also *Mathematical Papers, Vol. I.* Chelsea, New York, 1975, 495–500.

[14] —— The groups defined for a general field by the rotation groups. *University of Chicago Decennial Publications 9* (1902), 35–51; also *Mathematical Papers, Vol. I.* Chelsea, New York, 1975, 397–411.

[15] —— On the group defined for any given field by the multiplication table of any given finite group. *Trans. Amer. Math. Soc. 3* (1902), 285–301; also *Mathematical Papers, Vol. II.* Chelsea, New York, 1975, 75–91.

[16] —— On the groups defined for an arbitrary field by the multiplication table of certain finite groups. *Proc. London Math. Soc. 35* (1903), 68–80; also *Mathematical Papers, Vol. VI.* Chelsea, New York, 1983, 176–188.

[17] —— Modular theory of group-matrices. *Trans. Amer. Math. Soc. 8* (1907), 389–398; also *Mathematical Papers, Vol. II.* Chelsea, New York, 1975, 251–260.

[18] —— Modular theory of group characters. *Bull. Amer. Math. Soc.* (2) *13* (1907), 477–488; also *Mathematical Papers, Vol. IV.* Chelsea, New York, 1975, 535–546.

[19] FORMANEK, E. and D. SIBLEY. The group determinant determines the group. *Proc. Amer. Math. Soc. 112* (1991), 649–656.

[20] FROBENIUS, F.G. Über vertauschbare Matrizen. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1896), 601–614; also *Gesammelte Abhandlungen, Band II.* Springer-Verlag, New York, 1968, 705–718.

[21] —— Über Gruppencharaktere. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1896), 985–1021; also *Gesammelte Abhandlungen, Band III.* Springer-Verlag, New York, 1968, 1–37.

[22] —— Über die Primfactoren der Gruppendeterminante. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1896), 1343–1382; also *Gesammelte Abhandlungen, Band III.* Springer-Verlag, New York, 1968, 38–77.

[23] —— Über die Darstellung der endlichen Gruppen durch lineare Substitutionen. *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin* (1897), 944–1015; also *Gesammelte Abhandlungen, Band III.* Springer Verlag, New York, 1968, 82–103.

[24] HAWKINS, T. The origins of the theory of group characters. *Arch. Hist. Exact Sci. 7* (1971), 142–170.

[25] —— Hypercomplex numbers, Lie groups, and the creation of group representation theory. *Arch. Hist. Exact Sci. 8* (1972), 243–287.

[26] —— New light on Frobenius' creation of the theory of group characters. *Arch. Hist. Exact Sci. 12* (1974), 217–243.

[27] —— The creation of the theory of group characters. *Rice University Studies 64* (1978), 57–71.

[28] HOEHNKE, H.-J. and K. W. JOHNSON. The 1-, 2-, and 3-characters determine a group. *Bull. Amer. Math. Soc. (N.S.) 27* (1992), 243–245.

[29] HOEHNKE, H.-J. and K. W. JOHNSON. k-characters and group invariants. *Comm. Algebra 26* (1998), 1–27.

[30] JOHNSON, K. W. On the group determinant. *Math. Proc. Cambridge Philos. Soc. 109* (1991), 299–311.

[31] JOHNSON, K. W. and S. K. SEHGAL. The 2-characters of a group and the group determinant. *European J. Combin. 16* (1995), 632–631.

[32] LAM, T. Y. Representations of finite groups: A hundred years. *Notices Amer. Math. Soc. 45* (1998), pp. 361–372, 465–474.

[33] —— A theorem of Burnside on matrix rings. *Amer. Math. Monthly 105* (1998), 651–653.

[34] LANG, S. *Cyclotomic Fields I and II.* Springer-Verlag, New York, 1990.

[35] LEDERMANN, W. The origin of group characters. *J. Bangladesh Math. Soc. 1* (1981), 35–43.

[36] SATAKE, I. *Linear Algebra.* Marcel Dekker, New York, 1975.

[37] SPOTTISWOODE, W. Elementary theorems relating to determinants. *J. Reine Angew. Math. 51* (1856), 209–271, 328–381.

[38] van der WAERDEN, B. L. *A History of Algebra.* Springer-Verlag, New York, 1985.

(Reçu le 27 avril 1998)

Keith Conrad

Department of Mathematics,
Ohio State University,
Columbus, OH 43210-1174
U. S. A.
e-mail: kconrad@math.ohio-state.edu