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A DIOPHANTINE EQUATION INVOLVING FIFTH POWERS

by Ajai CHOUDHRY

ABSTRACT. The paper provides a parametric solution of the diophantine equation $aX_1^5 + bX_2^5 + cX_3^5 + dX_4^5 = aY_1^5 + bY_2^5 + cY_3^5 + dY_4^5$, where a, b, c, d are arbitrary non-zero integers.

While several parametric solutions of the diophantine equation

$$X_1^5 + X_2^5 + X_3^5 + X_4^5 = Y_1^5 + Y_2^5 + Y_3^5 + Y_4^5$$

are known [1,2,3], the diophantine equation

$$(1) \quad aX_1^5 + bX_2^5 + cX_3^5 + dX_4^5 = aY_1^5 + bY_2^5 + cY_3^5 + dY_4^5$$

has not been considered earlier. In this paper we give a parametric solution of (1) when a, b, c, d are arbitrary non-zero integers.

To solve (1), we write

$$(2) \quad \begin{cases} X_1 = m_1 p_1 u + m_2 v \alpha, & X_2 = m_1 q_1 u + m_2 v \beta, \\ X_3 = n_1 p_2 u + n_2 v \alpha, & X_4 = n_1 q_2 u + n_2 v \beta, \\ Y_1 = m_1 r_1 u + m_2 v \alpha, & Y_2 = m_2 v \beta, \\ Y_3 = n_1 r_2 u + n_2 v \alpha, & Y_4 = n_2 v \beta, \end{cases}$$

where $m_1, n_1, p_1, q_1, r_1, m_2, n_2, p_2, q_2, r_2, u, v, \alpha, \beta$ are arbitrary. Substituting these values in (1), we get an equation which may be written as

$$(3) \quad \sum_{i=1}^5 \binom{5}{i} u^i v^{5-i} \left[m_1^i m_2^{5-i} \{ a(p_1^i - r_1^i) \alpha^{5-i} + b q_1^i \beta^{5-i} \} \right. \\ \left. + n_1^i n_2^{5-i} \{ c(p_2^i - r_2^i) \alpha^{5-i} + d q_2^i \beta^{5-i} \} \right] = 0.$$

We now choose $p_1, q_1, r_1, p_2, q_2, r_2$ as follows :

$$(4) \quad \begin{cases} p_1 = a\alpha^5 - b\beta^5, & q_1 = 2a\alpha^4\beta, & r_1 = a\alpha^5 + b\beta^5, \\ p_2 = -c\alpha^5 + d\beta^5, & q_2 = -2c\alpha^4\beta, & r_2 = -c\alpha^5 - d\beta^5. \end{cases}$$

With these values of $p_1, q_1, r_1, p_2, q_2, r_2$ (and m_1, n_1, m_2, n_2 arbitrary), we find that the coefficients of uv^4 and u^2v^3 in equation (3) become zero. The coefficient of u^3v^2 in (3) will also vanish if

$$(5) \quad abm_1^3m_2^2(a^2\alpha^{10} - b^2\beta^{10}) - cdn_1^3n_2^2(c^2\alpha^{10} - d^2\beta^{10}) = 0.$$

We accordingly choose

$$(6) \quad \begin{cases} m_1 = n_2, & n_1 = m_2, \\ m_2 = ab(a^2\alpha^{10} - b^2\beta^{10}), & n_2 = cd(c^2\alpha^{10} - d^2\beta^{10}), \end{cases}$$

so that (5) is satisfied. Now, equation (3) has only the terms involving u^4v and u^5 and it is readily solved to give the following solution for u, v :

$$(7) \quad \begin{cases} u = -20a^2b\alpha^6m_1^4m_2(a^2\alpha^{10} - b^2\beta^{10}) \\ \quad \quad \quad - 20c^2d\alpha^6n_1^4n_2(c^2\alpha^{10} - d^2\beta^{10}), \\ v = abm_1^5(11a^4\alpha^{20} - 10a^2b^2\alpha^{10}\beta^{10} - b^4\beta^{20}) \\ \quad \quad \quad - cdn_1^5(11c^4\alpha^{20} - 10c^2d^2\alpha^{10}\beta^{10} - d^4\beta^{20}). \end{cases}$$

Thus, a parametric solution of equation (1) in terms of the parameters α and β is given by (2), where m_1, n_1, m_2, n_2 are defined by (6), $p_1, q_1, r_1, p_2, q_2, r_2$ by (4), and u, v by (7).

As a numerical example, when $a = 1, b = 2, c = 3, d = 6$, taking $\alpha = 1, \beta = 2$, we get the following solution of (1):

$$\begin{aligned} (X_1, X_2, X_3, X_4) &= (1955587, 2963474, 121184667, 242404434), \\ (Y_1, Y_2, Y_3, Y_4) &= (1022467, 2992634, 121219227, 242403354). \end{aligned}$$

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