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We now choose $p_1, q_1, r_1, p_2, q_2, r_2$ as follows :

$$(4) \quad \begin{cases} p_1 = a\alpha^5 - b\beta^5, & q_1 = 2a\alpha^4\beta, & r_1 = a\alpha^5 + b\beta^5, \\ p_2 = -c\alpha^5 + d\beta^5, & q_2 = -2c\alpha^4\beta, & r_2 = -c\alpha^5 - d\beta^5. \end{cases}$$

With these values of $p_1, q_1, r_1, p_2, q_2, r_2$ (and m_1, n_1, m_2, n_2 arbitrary), we find that the coefficients of uv^4 and u^2v^3 in equation (3) become zero. The coefficient of u^3v^2 in (3) will also vanish if

$$(5) \quad abm_1^3m_2^2(a^2\alpha^{10} - b^2\beta^{10}) - cdm_1^3n_2^2(c^2\alpha^{10} - d^2\beta^{10}) = 0.$$

We accordingly choose

$$(6) \quad \begin{cases} m_1 = n_2, & n_1 = m_2, \\ m_2 = ab(a^2\alpha^{10} - b^2\beta^{10}), & n_2 = cd(c^2\alpha^{10} - d^2\beta^{10}), \end{cases}$$

so that (5) is satisfied. Now, equation (3) has only the terms involving u^4v and u^5 and it is readily solved to give the following solution for u, v :

$$(7) \quad \begin{cases} u = -20a^2b\alpha^6m_1^4m_2(a^2\alpha^{10} - b^2\beta^{10}) \\ \quad \quad \quad - 20c^2d\alpha^6n_1^4n_2(c^2\alpha^{10} - d^2\beta^{10}), \\ v = abm_1^5(11a^4\alpha^{20} - 10a^2b^2\alpha^{10}\beta^{10} - b^4\beta^{20}) \\ \quad \quad \quad - cdm_1^5(11c^4\alpha^{20} - 10c^2d^2\alpha^{10}\beta^{10} - d^4\beta^{20}). \end{cases}$$

Thus, a parametric solution of equation (1) in terms of the parameters α and β is given by (2), where m_1, n_1, m_2, n_2 are defined by (6), $p_1, q_1, r_1, p_2, q_2, r_2$ by (4), and u, v by (7).

As a numerical example, when $a = 1, b = 2, c = 3, d = 6$, taking $\alpha = 1, \beta = 2$, we get the following solution of (1):

$$\begin{aligned} (X_1, X_2, X_3, X_4) &= (1955587, 2963474, 121184667, 242404434), \\ (Y_1, Y_2, Y_3, Y_4) &= (1022467, 2992634, 121219227, 242403354). \end{aligned}$$

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