

Zeitschrift: L'Enseignement Mathématique
Band: 45 (1999)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: COUNTING PATHS IN GRAPHS

Kurzfassung

Autor: Bartholdi, Laurent

DOI: <https://doi.org/10.5169/seals-64442>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 20.11.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

COUNTING PATHS IN GRAPHS

by Laurent BARTHOLDI

ABSTRACT. We give a simple combinatorial proof of a formula that extends a result by Grigorchuk [Gri78a, Gri78b] relating cogrowth and spectral radius of random walks. Our main result is an explicit equation determining the number of ‘bumps’ on paths in a graph: in a d -regular (not necessarily transitive) non-oriented graph let the series $G(t)$ count all paths between two fixed points weighted by their length t^{length} , and $F(u, t)$ count the same paths, weighted as $u^{\text{number of bumps}} t^{\text{length}}$. Then one has

$$\frac{F(1-u, t)}{1-u^2 t^2} = \frac{G\left(\frac{t}{1+u(d-u)t^2}\right)}{1+u(d-u)t^2}.$$

We then derive the circuit series of ‘free products’ and ‘direct products’ of graphs. We also obtain a generalized form of the Ihara-Selberg zeta function [Bas92, FZ98].

1. INTRODUCTION

Let $\Gamma = \mathbf{F}_S/N$ be a group generated by a finite set S , where \mathbf{F}_S denotes the free group on S . Let f_n be the number of elements of the normal subgroup N of \mathbf{F}_S whose minimal representation as words in $S \cup S^{-1}$ has length n ; let g_n be the number of (not necessarily reduced) words of length n in $S \cup S^{-1}$ that evaluate to 1 in Γ ; and let $d = |S \cup S^{-1}| = 2|S|$. The numbers

$$\alpha = \limsup_{n \rightarrow \infty} \sqrt[n]{f_n}, \quad \nu = \frac{1}{d} \limsup_{n \rightarrow \infty} \sqrt[n]{g_n}$$

are called the *cogrowth* and *spectral radius* of (Γ, S) . The Grigorchuk Formula [Gri78b] states that

$$(1.1) \quad \nu = \begin{cases} \frac{\sqrt{d-1}}{d} \left(\frac{\alpha}{\sqrt{d-1}} + \frac{\sqrt{d-1}}{\alpha} \right) & \text{if } \alpha > \sqrt{d-1}, \\ \frac{2\sqrt{d-1}}{d} & \text{else.} \end{cases}$$