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A p-ADIC L-FUNCTION OF TWO VARIABLES

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ABSTRACT. For p prime and χ a primitive Dirichlet character, we derive a p-adic function $L_p(s,t;\chi)$, where $t \in \mathbf{C}_p$, $|t|_p \leq 1$, and $s \in \mathbf{C}_p$, $|s-1|_p < |p|_p^{1/(p-1)}|q|_p^{-1}$, $s \neq 1$ if $\chi = 1$, with $q = 4$ if $p = 2$ and $q = p$ if $p > 2$, that interpolates the values

$$L_p(1-n,t;\chi) = -\frac{1}{n}\left(B_{n,\chi_n}(qt) - \chi_n(p)p^{n-1}B_{n,\chi_n}(p^{-1}qt)\right),$$

for $n \in \mathbf{Z}$, $n \geq 1$. Here $B_{n,\chi}(t)$ is the n^{th} generalized Bernoulli polynomial associated with the character χ, and $\chi_n = \chi\omega^{-n}$, where ω is the Teichmüller character. This function is then a two-variable analogue of the p-adic L-function $L_p(s;\chi)$, where $s \in \mathbf{C}_p$, $|s-1|_p < |p|_p^{1/(p-1)}|q|_p^{-1}$, $s \neq 1$ if $\chi = 1$, in that this function satisfies $L_p(s,0;\chi) = L_p(s;\chi)$. In addition to deriving this function, we establish several properties and applications of $L_p(s,t;\chi)$.

1. INTRODUCTION

Given a primitive Dirichlet character χ, having conductor f_χ (see Section 2 for definitions), the Dirichlet L-function associated with χ is defined by

$$L(s;\chi) = \sum_{b=1}^{\infty} \frac{\chi(b)}{b^s},$$

where $s \in \mathbf{C}$, $\Re(s) > 1$. This function can be continued analytically to the entire complex plane, except for a simple pole at $s = 1$ when $\chi = 1$, in which case we have the Riemann zeta function, $\zeta(s) = L(s;1)$. It is believed that the analysis of Dirichlet L-functions began with Euler's study of $\zeta(s)$, in which he considered the function only for real values of s. It was Riemann

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