

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 46 (2000)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE SPECTRAL MAPPING THEOREM, NORMS ON RINGS, AND RESULTANTS
Autor: Laksov, D. / Svensson, L. / Thorup, A.
Kurzfassung
DOI: <https://doi.org/10.5169/seals-64805>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 22.12.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

THE SPECTRAL MAPPING THEOREM,
NORMS ON RINGS, AND RESULTANTS

by D. LAKSOV, L. SVENSSON and A. THORUP

ABSTRACT. We give a short, simple and self-contained proof of the Spectral Mapping Theorem for matrices with entries in an arbitrary commutative ring. The result is placed in the wider framework of norms on algebras. It is shown that the Spectral Mapping Theorem follows from a uniqueness result for norms on polynomial rings in one variable. The results are used to generalize classical formulas for the resultant of polynomials.

1. INTRODUCTION

It is a well-known and useful result in spectral theory of complex finite dimensional vector spaces that if the characteristic polynomial of an $n \times n$ -matrix M splits as $P_M(t) = \det(tI_n - M) = \prod_{i=1}^n (t - \lambda_i)$ then, for any polynomial $F(x)$, we have that $\det(tI_n - F(M)) = \prod_{i=1}^n (t - F(\lambda_i))$. We call this result the Spectral Mapping Theorem, because it is similar to the Spectral Mapping Theorem for Banach algebras. Many proofs of the result for complex finite dimensional vector spaces are known, most of them based upon transforming the matrix into triangular form (see [B2], §5, Proposition 10, p.36), or using the Jordan canonical form for the matrix (see [L], Chapter XIV, §3, Theorem 3.10, p.566). The Theorem and its proofs are easily generalized to arbitrary fields, and therefore to integral domains. In our work on parameter spaces in algebraic geometry ([L-S], [S1], [S2]) we needed a generalization of the Spectral Mapping Theorem to matrices with entries in arbitrary commutative rings with unity. The only reference we could find to such a generalization was [L], Chapter XIV, §3, Theorem 3.10, p.566, where a proof is deduced from the theory of integral ring extensions. The difficult part of the proof is dismissed with the phrase “This is obvious to the reader who read the chapter on integral ring extensions, and the reader