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THE SPECTRAL MAPPING THEOREM, NORMS ON RINGS, AND RESULTANTS

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ABSTRACT. We give a short, simple and self-contained proof of the Spectral Mapping Theorem for matrices with entries in an arbitrary commutative ring. The result is placed in the wider framework of norms on algebras. It is shown that the Spectral Mapping Theorem follows from a uniqueness result for norms on polynomial rings in one variable. The results are used to generalize classical formulas for the resultant of polynomials.

1. INTRODUCTION

It is a well-known and useful result in spectral theory of complex finite dimensional vector spaces that if the characteristic polynomial of an $n \times n$-matrix M splits as $P_M(t) = \det(tI_n - M) = \prod_{i=1}^{n}(t - \lambda_i)$ then, for any polynomial $F(x)$, we have that $\det(tI_n - F(M)) = \prod_{i=1}^{n}(t - F(\lambda_i))$. We call this result the Spectral Mapping Theorem, because it is similar to the Spectral Mapping Theorem for Banach algebras. Many proofs of the result for complex finite dimensional vector spaces are known, most of them based upon transforming the matrix into triangular form (see [B2], §5, Proposition 10, p. 36), or using the Jordan canonical form for the matrix (see [L], Chapter XIV, §3, Theorem 3.10, p. 566). The Theorem and its proofs are easily generalized to arbitrary fields, and therefore to integral domains. In our work on parameter spaces in algebraic geometry ([L-S], [S1], [S2]) we needed a generalization of the Spectral Mapping Theorem to matrices with entries in arbitrary commutative rings with unity. The only reference we could find to such a generalization was [L], Chapter XIV, §3, Theorem 3.10, p. 566, where a proof is deduced from the theory of integral ring extensions. The difficult part of the proof is dismissed with the phrase "This is obvious to the reader who read the chapter on integral ring extensions, and the reader