Zeitschrift: L'Enseignement Mathématique
Band: 46 (2000)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THREE REMARKS ON GEODESIC DYNAMICS AND FUNDAMENTAL GROUP

Bibliographie

Autor: Gromov, Mikhaïl
DOI: https://doi.org/10.5169/seals-64809

Nutzungsbedingungen
Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation
L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use
The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 19.11.2024

ETH-Bibliothek Zürich, E-Periodica, https://www.e-periodica.ch

via the "asymptotic stretch" of $f_*: \pi_1 \to \pi_1$ with respect to the word metric in π_1 (see [Bow]). But this is not sharp even for linear automorphisms of tori T^n, where the entropy is expressed by the "k-dimensional stretch" on H_1 for some $k \leq n$ that equals the spectral radius of f_* on H_k. Such k-stretch can be defined, in general, in terms of $f_*: \pi_1(S) \to \pi_1(S)$ and the classifying map $S \to K(\pi_1, 1)$ (refining the spectral radius of f_* on H^k coming from $K(\pi_1, 1)$), but my obvious "proof" of the lower bound on the entropy by this k-stretch missed a hidden trap. This was also overlooked in [Ma$_3$] (for $f_*: H_1 \to H_1$, where a proper identification of the "k-stretch" with the spectral radius needs extra work), as was pointed out to me much later by David Fried. (The difficulty already appears for closed subsets S in the torus T^n invariant under linear automorphisms f of T^n, where one wishes to estimate the entropy of $f|S$ in terms of f_* acting on the *spectral* cohomology of S coming from T^n. On the other hand, the case of $T^n \to T^n$ is settled in [Mi-Pr].)

APPENDIX 6. Probably, the recent progress in Nielsen theory allows a description of the cases, where card(Fix$f$) is well controlled from below by some twisted Lefschetz number (see [Fel]).

APPENDIX 7. Nothing interesting to say.

APPENDIX 8. Minima of geometric functionals related to the logical complexity have been studied in depth by A. Nabutovski (see [Na] and references therein). Yet I do not feel ready yet to write this Appendix. For example, I do not see what is the actual influence of a suitable (?) complexity measure of $\pi_1(V)$ on the Plateau problem in V.

BIBLIOGRAPHY

[B-G-S] BALLMANN, W., M. GROMOV and V. SCHROEDER. *Manifolds of Non-Positive Curvature.* Birkhäuser, Boston, 1985.

[Bow] BOWEN, R. Entropy and the fundamental group. In: *The Structure of Attractors in Dynamical Systems. (Proc. Conf., North Dakota State Univ., Fargo, N.D. (1977))*, 21–29. Lecture Notes in Math., 668, Springer, Berlin, 1978.

[Br-Ha] BRIDSON, M. R. and A. HAEFLIGER. *Metric Spaces of Non-Positive Curvature.* Grundlehren der mathematischen Wissenschaften, Vol. 319. Springer, Berlin, 1999.

[Din] DINABURG, E. I. A connection between various entropy characterizations of dynamical systems (Russian). *Izv. Akad. Nauk SSSR Ser. Mat. 35* (1971), 324–366.

[Fa-Jo] FARRELL, F. T. and L. E. JONES. A topological analogue of Mostow's rigidity theorem. *J. Amer. Math. Soc. 2* (1989), 257–370.

[Fel] FEL'SHTYN, A. Dynamical zeta functions, Nielsen theory and Reidemeister torsion. *Mem. Amer. Math. Soc. 147* (2000), no. 699.

[Gro$_1$] GROMOV, M. Hyperbolic manifolds, groups and actions. In: *Riemann Surfaces and Related Topics (Proc. of the 1978 Stony Brook Conference), Ann. Math. Studies 97* (1981), 183–213, Princeton University Press, Princeton.

[Gro$_2$] —— Positive curvature, macroscopic dimension, spectral gaps and higher signatures. In: *Functional Analysis on the Eve of the 21st Century.* Vol. II (In honor of the eightieth birthday of I. M. Gel'fand. *Proc. conf. Rutgers Univ., 1993.*) 1–213, Birkhäuser, Basel, 1996.

[Ma$_1$] MANNING, A. Topological entropy and the first homology group. In: *Dynamical Systems – Warwick 1974* (Proc. Sympos. Appl. Topology and Dynamical Systems, Univ. Warwick, Coventry, 1973/1974; presented to E. C. Zeeman on his fiftieth birthday), 185–190. Lecture Notes in Math., 468, Springer, Berlin, 1975.

[Ma$_2$] —— Topological entropy for geodesic flows. *Ann. of Math. (2) 110* (1979), 567–573.

[Ma$_3$] —— Toral automorphisms, topological entropy and the fundamental group. In: *Dynamical Systems, Vol. II – Warsaw*, 273–281. Astérisque *50*, 1977.

[Mi-Pr] MISIUREWICZ, M. and F. PRZYTYCKI. Entropy conjecture for tori. *Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. 25* (1977), 575–578.

[Na] NABUTOVSKY, A. Disconnectedness of sublevel sets of some Riemannian functionals. *Geom. Funct. Anal. 6* (1996), 703–725.

Mikhaïl Gromov

Institut des Hautes Études Scientifiques
35, route de Chartres
F-91440 Bures-sur-Yvette
France