Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 46 (2000)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: REPORT ON THE ICMI STUDY : «PERSPECTIVES ON THE TEACHING OF GEOMETRY FOR THE 21ST CENTURY»

Autor: VILLANI, Vinicio

DOI: https://doi.org/10.5169/seals-64811

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L'Enseignement Mathématique, t. 46 (2000), p. 411–415

COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

(THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

A REPORT ON THE ICMI STUDY: «PERSPECTIVES ON THE TEACHING OF GEOMETRY FOR THE 21ST CENTURY»

by Vinicio VILLANI

In 1994 *L'Enseignement Mathématique* (vol. 40, pp. 345–357) published the Discussion Document for an ICMI Study entitled *"Perspectives on the Teaching of Geometry for the 21st Century"*. The next step was an international conference held in Catania in 1995. Three years later, a volume with the same title was published in the *ICMI Studies* series (see [1]).

Recently the Editors of *L'Enseignement Mathématique* asked me to report briefly on the main outcomes of that study. The task of summarizing in one volume the richness and variety of the contributions on the theme under discussion had been already very difficult. But it is almost impossible to compress a presentation of the whole volume into three or four pages.

All that I can do is give a glimpse of its content, through short quotations of some sentences from each chapter. These sentences were selected nearly at random, with the only concern that they should be intelligible even outside their natural context.

I am confident that this kind of presentation may be at least sufficient to raise in some readers the curiosity to have a look at the volume.

From the INTRODUCTION (Contributions by *C. Mammana and V. Villani*):

The perfection of Euclid's treatise became a model and a prototype for a rational systematization of all fields of knowledge. [...] On the other hand, this same perfection of Euclid's treatise inhibited further progress in geometry itself, resulting in a 'freezing' of geometric knowledge for almost 2000 years within the Euclidean scheme.

It is therefore not surprising that, after many centuries of general stagnation, original ideas in geometric research arose from stimuli coming from outside Euclidean geometry: during the fifteenth century from studies in an artistic milieu about perspective (Piero della Francesca, Leon Battista Alberti), in the seventeenth century from a melding of geometry and algebra (Descartes) and towards the end of the eighteenth century from a systematic study of the methods of representation of 3-dimensional objects through drawings, i.e. by descriptive geometry (Monge). [...] One must wait until the nineteenth century in order to achieve an advance beyond Euclidean geometry, thanks to the discovery of non-Euclidean geometries (Gauss, Bolyai, Lobachevsky).

[...] Recently, it seems that we are experiencing a renewed interest in the visual aspects of geometry, but once more this research is growing mostly outside the milieu of the mathematical community. Just think about the achievements concerning *applications* of geometry in computer graphics, in image reconstruction and image processing, in pattern recognition and robotics. In all these research fields the most active departments are those of computer science, engineering, chemistry, and medicine.

From CHAPTER 1. GEOMETRY: PAST AND FUTURE (Contributions by *V. Lundsgaard Hansen, J. Malkevitch, A. Douady*):

Hyperbolic geometry in 2D, and also 3D, is an important cultural object. [...] But putting it in the curriculum, to be studied in any detail, would probably be unrealistic in view of the many constraints on the organization of teaching and of the epistemological nature of the question which motivates only a minority.

Yet another type of geometry is that on the sphere. [...] It gives a reasonably good model of the Earth, so it is culturally important. [...] This requires at least a good level in 3D Euclidean geometry and in trigonometry. Geometry on the sphere differs from Euclidean geometry by the fact that there are several geodesics which join two antipodal points. Except for that, it is technically very similar to hyperbolic geometry.

From CHAPTER 2. REASONING IN GEOMETRY (Contributions by *R. Hershkowitz, R. Duval, M. Bartolini Bussi and P. Boero, R. Lehrer and T. Romberg, R. Berthelot and M.H. Salin, K. Jones*):

For generations, geometry was taught as *the* context for teaching deductive reasoning. [...] The product – a written proof – was more important than the process of proving, and thus teaching tended to neglect both the visual geometrical context [...] and the learner. Nowadays, research and development effort is being invested in order to create innovative learning environments that still regard deductive reasoning as a basic element to be learned. However, these learning environments try to take into account the pupil's point of view by designing learning situations which help pupils *feel* an intrinsic need for explanations, and thus provide the invitation to appreciate the strength of deductive justification as an explanatory tool, or even to attempt to produce them.

From CHAPTER 3. GEOMETRY IN OUR WORLD (Contributions by *J. Malkevitch, W. Meyer, P. Legisa*):

Although historically geometry has proved useful in solving a variety of problems in everyday life and physical science [...], an unexpected recent surprise has been the extent to which geometric thinking has been of value in support of rapidly emerging new technologies. [Several challenging examples from robotics, medical imaging, telecommunications, image manipulation and processing, operations research follow.]

From CHAPTER 4. COMPUTER TECHNOLOGY AND THE TEACHING OF GEOMETRY (Contributions by *I. Osta, C. Laborde, C. Hoyles and K. Jones, K-D. Graf and B. Hodgson*):

There is general agreement among educators and teachers that the computer can provide a valuable means for visualizing geometric situations. Software packages of various types use animation capabilities to provide ways for constructing, moving and rotating configurations, for observing them under various angles, and for modifying some of their features. These demonstrative functions lead to a more functional role of the computer as an explorative tool, making intuition, construction, and spatial sense more important factors, but also providing ways to link them to the theoretical aspects.

[The chapter is divided into four sections.] In Section II the question is posed, whether the introduction of dynamic geometry systems will promote the transition from informal proof to formal proof in mathematics, or whether it will make it even harder. Will computer use assist pupils in developing a conceptual framework for proof, or will it be seen to replace any need for a proof?

[...] Through the use of computer graphics we offer efficient new modes for the inspection of mathematical processes, like transforming a figure pointwise or iterating a transformation (in a simulation of a kaleidoscope, for instance). Traditional actions with ruler and compass would be extremely inefficient for these purposes.

From CHAPTER 5. GEOMETRY IN THE CLASSROOM (Contributions by *R. Douady and B. Parzysz*):

Geometry allows bringing into play intimate convictions, coming from familiarity with the environment in which we live or with visual perception. This assistance lets us enter the problem with some ideas. But, at the same time, it can be misleading or take us to a dead end. It is then essential to have other tools at our disposal in order to allow the search to progress, to ensure the control of reasoning and guarantee the consistency or, at least, to trace inconsistencies, incompatibilities, even if some questioning is left. *Algebra* can be a model-framework and can provide tools adapted for dealing with the problem.

From CHAPTER 6. THE EVOLUTION OF GEOMETRY SINCE 1900 (Contributions by *H. B. Griffiths, M. Galuzzi, M. Neubrand and C. Laborde*):

One practice needs investigation because it seems to be only superficially efficient – that of teaching Geometry alone for two or three semesters, as in High Schools that follow the US custom. This practice assumes (as do many non-teachers) that once taught, a topic never needs mention again, as if pupils had computers for memories; and of course [...] ignores the need for a spiral approach through the levels, and to show the interplay between various parts of mathematics.

[...] Experience from the 1960's shows that if teachers are simply told what to do by hierarchies of inspectors, who then say "Now get on with the job", then disaster will ensue.

From CHAPTER 7. CHANGES AND TRENDS IN GEOMETRY CURRICULA (Contributions by *V. Lundsgaard Hansen, C. E. Vasco, G. K. Gholam, J. Tocki and S. Turnau, S. Tang and F. Zhang, M. Neubrand*):

It seems an impossible task to suggest a core curriculum in geometry with the hope of having it implemented in all countries. As already indicated, it is more fruitful in that respect to suggest a few main guidelines for the teaching of geometry throughout the school system. We can offer two such main guidelines, namely:

- the study of magnitudes (estimation, calculation, measuring),
- the study of plane representations of spatial objects.

From CHAPTER 8. ASSESSMENT IN GEOMETRY (Contributions by *M. Niss, J. Pegg, A. Gutiérrez and P. Huerta*):

[...] What is traditionally in focus is learners' knowledge of (geometric) facts [...], mastery of standard methods and techniques [...], performance of standard applications. [...] It is much rarer to encounter assessment objects such as:

- visualisation;
- open-ended problem solving;
- geometric modelling of complex extra-mathematical situations;
- rigorous and heuristic reasoning;
- generation and exploration of hypotheses;
- explaining the structure of a geometric theory;
- establishing links between different geometric topics;
- interpreting an abstract geometric theory in relation to a specific object domain.

From CHAPTER 9. TEACHER QUALIFICATIONS AND THE EDUCATION OF TEACHERS (contributions by *M. Niss*):

[...] If the problem with the geometrical education of primary school teachers lies mainly in the absence of knowledge of geometrical theory, the problem tends to be the reverse with teachers of secondary and tertiary levels. If they have any geometrical education, it is likely to be predominantly theoretical, whereas the other dimensions of geometry, equally important in the context of education, are usually greatly under-represented both in pre-service training and in in-service programmes. Of course, it is important that teachers have a solid and varied background in different species of theoretical geometry. [...] But in order to be able to endow their teaching of geometry with richness in quality and perspective, and to exercise it with autonomy and self-confidence, their knowledge and views of the discipline need to encompass also other dimensions of geometry than the purely theoretical ones.

From CHAPTER 10. THE WAY AHEAD (Contributions by *V. Villani on behalf of the International Program Committee*):

In selecting the theorems to be proved, non-intuitive, surprising statements should be privileged, in order to show the power of reasoning over mere physical experience (*e.g.* the sum of the angles of a triangle, Pythagoras' theorem, properties of the Euler line in a triangle, incommensurability of side and diagonal of a square.

[...] Finally, 'theorem proving' should not be confined to geometry. Even in arithmetic, algebra and probability, there are many opportunities for proving easy but significant statements. Of course, the style of proofs is rather different in these contexts. What is peculiar to geometric proofs is the role of visual intuition, which is why it is so difficult to give a totally rigorous proof of anything in geometry (and why New Math turned to algebra instead).

REFERENCES

[1] *Perspectives on the Teaching of Geometry for the 21st Century.* An ICMI Study. Edited by C. Mammana and V. Villani. Kluwer Acad. Publ., 1998.

(Reçu le 16 novembre 2000)

Vinicio Villani
Dipartimento di Matematica
Università di Pisa
Via Buonarroti 2
I-56127 Pisa
Italy
e-mail: villani@dm.unipi.it

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