

Zeitschrift: L'Enseignement Mathématique
Band: 46 (2000)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE SATURATION CONJECTURE (AFTER A. KNUTSON AND T. TAO)

Bibliographie

Autor: BUCH, Anders Skovsted / Fulton, William

DOI: <https://doi.org/10.5169/seals-64794>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 19.11.2024

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

$\lambda^{(1)} \supset \lambda^{(2)} \supset \dots \supset \lambda^{(n)} \supset \emptyset$, and from these partitions one successively fills in the entries in the northwest to southeast diagonal rows of the hive; the rhombus inequalities (1)–(3) are automatically satisfied.

To make the story complete, we recall why such contratableaux correspond to Littlewood-Richardson skew tableaux, using standard results about tableaux, as in [5]. However, it may be pointed out that these contratableaux are at least as easy to produce and enumerate as the more classical skew tableaux. First, the condition that $w(T) \cdot w(U(\mu))$ is a reverse lattice word, given that the number of times i occurs in T is $\nu_i - \mu_i$, is equivalent to asserting that $w(T) \cdot w(U(\mu))$ is Knuth equivalent to $w(U(\nu))$ [5, §5.2]. The rectification R of a contratableau T of shape λ is easily seen to be a tableau of shape λ , and with the same property that $w(R) \cdot w(U(\mu))$ is Knuth equivalent to $w(U(\nu))$. The correspondence between tableaux and contratableaux of shape λ is a bijection, by reversing the rectification process.

Now the condition that $w(R) \cdot w(U(\mu))$ be Knuth equivalent to $w(U(\nu))$ is equivalent to the condition that $R \cdot U(\mu) = U(\nu)$ in the plactic monoid of tableaux [5, §2.1]. It is easy to see, from the definition of multiplying tableaux by column bumping entries of the first tableau into the second [5, §A.2], that if R and S are tableaux with $R \cdot S = U(\beta)$, then S must be equal to $U(\alpha)$ for some partition α . This gives a correspondence between the set of tableaux R that we are looking at and the set of pairs (R, S) with R of shape λ , S of shape μ , whose product is the tableau $U(\nu)$. There is a standard construction [5, §5.1] between these pairs and the set of skew tableau on the shape ν/λ of content μ whose word is a reverse-lattice word.

REFERENCES

- [1] BERENSTEIN, A. D. and A. V. ZELEVINSKY. Triple multiplicities for $\mathfrak{sl}(r+1)$ and the spectrum of the exterior algebra of the adjoint representation. *J. Algebraic Combin.* 1 (1992), 7–22.
- [2] BERTRAM, A., I. CIOCAN-FONTANINE and W. FULTON. Quantum multiplication of Schur polynomials. *J. Algebra* 219 (1999), no. 2, 728–746.
- [3] BUCH, A. S. and W. FULTON. Chern class formulas for quiver varieties. *Invent. Math.* 135 (1999), no. 3, 665–687.
- [4] CARRÉ, C. The rule of Littlewood-Richardson in a construction of Berenstein-Zelevinsky. *Internat. J. Algebra Comput.* 1 (1991), 473–491.
- [5] FULTON, W. Young tableaux. London Mathematical Society Student Texts, vol. 35. Cambridge University Press, 1997.
- [6] ———. Eigenvalues of sums of Hermitian matrices (after A. Klyachko). *Astérisque* (1998), no. 225, 225–269, Séminaire Bourbaki, Vol. 1997/98.

- [7] GOODMAN, F.M. and H. WENZL. Littlewood-Richardson coefficients for Hecke algebras at roots of unity. *Adv. Math.* 82 (1990), 244–265.
- [8] HORN, A. Eigenvalues of sums of Hermitian matrices. *Pacific J. Math.* 12 (1962), 225–241.
- [9] KLYACHKO, A. A. Stable bundles, representation theory and Hermitian operators. *Selecta Math. (N.S.)* 4, 1998, no. 3, 419–445.
- [10] KNUTSON, A. and T. TAO. The honeycomb model of the Berenstein-Zelevinsky polytope I: Klyachko's saturation conjecture. *J. Amer. Math. Soc.* 12, (1999), no. 4, 1055–1090.
- [11] STURMFELS, B. and R.R. THOMAS. Variation of cost functions in integer programming. *Math. Programming* 77 (1997), no. 3, Ser. A, 357–387.
- [12] ZELEVINSKY, A. Littlewood-Richardson semigroups. *MSRI Preprint*, 1997-044.

(Reçu le 5 janvier 1999)

Anders Skovsted Buch

Massachusetts Institute of Technology
Building 2, Room 248
77 Massachusetts Avenue
Cambridge, MA 02139
USA
e-mail: abuch@math.mit.edu